

Research on Equilibrium Decision Closed-loop Supply Chain Super-network Considering Consumer Channel Preferences Led by Manufacturers

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Abstract

In order to study how consumers' preferences for shopping through online channels, and the intensity of competition between online and traditional channels affect the decision-making and optimal goals of companies at all levels in a closed loop supply chain network dominated by manufacturers, this paper establishes a closed-loop supply chain super-network equilibrium model consisting of multiple suppliers, manufacturers, retailers, demand markets and recyclers. In this model, companies at all levels of the supply chain aim to maximize profits. This paper uses the method of equilibrium theory and variational inequality to analyze the decision-making behaviors and optimal targets of enterprises at various levels, and solves them through an improved projection gradient algorithm. Finally, this paper uses numerical examples to analyze the impact of consumer channel preferences and the intensity of competition between channels on the transaction volume, profits of various layers of enterprises, and the prices of demand for products purchased through online and traditional channels.

Keywords

Closed-loop supply chain; manufacturer-led; channel preference; super-network equilibrium; variational inequality .

1. Introduction

With the advancement of the times and the development of the Internet, e-commerce companies in china have sprung up rapidly. Many manufacturers have broken the traditional business model, opened up online direct sales channels, and provided consumers with new, convenient, and trendy Shopping methods. Therefore, a dual-channel supply chain with manufacturers as the main body has been formed. Although this emerging shopping method has brought different shopping experiences to consumers, brought considerable profits to some enterprises, and also promoted the development of the logistics industry, the operation of online direct sales channels has brought unprecedented opportunities to traditional channels. And this shock reduced the sales volume of traditional channels, damaged the interests of retailers, and was not conducive to the stable development of the entire closed-loop supply chain. Therefore, how to coordinate the supply chain network to achieve equilibrium has become a very important issue. Regarding the coordination of dual-channel supply chains, Zhang considered the risk of return and pricing differences and studied the pricing strategies of manufacturers under different market demands (Zhang Xuelong, Wu Doudou,etc,2018) [1].This paper solved the problem of supply chain channel operation combination in the case of product experience differences between physical channels and network channels(Zhou Jianheng, Wang Qi,2017) [2].The paper studied the two-tier supply chain network composed of manufacturers and retailers, and the impact of retailers opening network channels on the supply chain (Kong Zaojie and Li Zuyi,2017)[3]. Yu studied the supply chain equilibrium

between companies with offline and online sales channels (Yugang Yu, Xiaoya Han, etc, 2017)[4]. Sun also considered consumers' preferences for low carbon and channels, and analyzed the numerical examples to obtain the optimal carbon emission boundary of the supply chain (Sun Jianan and Xiao Zhongdong, 2015)[5]. In the research of supply chain network equilibrium, the paper considered the impact of consumer behavior in the online shopping supply chain system on the online shopping supply chain network equilibrium during the promotion of e-commerce companies, and constructed a market based on consumer behavior. Finally, the paper analyzed the best decision-making behavior of decision makers at all levels (Ye Zhou, Jue Zeng, etc, 2017)[6]. Hamdouch and others proposed a decentralized closed-loop supply chain network model, assuming that the demand for products and the corresponding returns are random and price-sensitive, and finally analyzed the optimal decision-making behavior of enterprises at all levels (Younes Hamdouch, Qiang Patrick Qiang, Kilani Ghoudi, 2017)[7]. The paper considered the loss caused by the interruption risk between the producer and the retailer, and studied the dynamic management of supply chain risk (Ma Jun, Dong Qiong and Yang Deli, 2015)[8]. As the country strongly advocates saving resources and recycling resources, more and more scholars attach importance to the research of closed-loop supply chain management. Cao Xiaogang and others constructed a closed-loop supply chain network composed of multiple suppliers, manufacturers, retailers, and consumer markets, and analyzed the impact of risk factors on the equilibrium results of the supply chain network (Cao Xiaogang, Zhen Benrong, etc, 2014)[9].

Most of the supply chain networks constructed by the above studies are made up of manufacturers and retailers, without taking into account other members of the supply chain, such as suppliers, demand markets, and recyclers. In addition, although the dual-channel supply chain coordination problem is considered, the consideration of consumer channel preferences has less impact on the closed-loop supply chain. In view of the above issues, this paper takes a closed-loop supply chain consisting of multiple suppliers, multiple manufacturers, multiple retailers, multiple demand markets, and multiple recyclers as the research object. When manufacturers open up network channels, considering consumers' preference for using online shopping, a closed-loop supply chain super-network equilibrium model was established. Then, a numerical example is conducted to discuss the influence of consumer channel preferences and inter-channel competition intensity factors on the transaction volume, demand price, and corporate profits of various layers of companies. Finally, this paper gives some suggestions for coordinating closed-loop supply chains.

2. Model Construction and Symbol Description

2.1. Model Building

We construct a closed-loop supply chain model consists of five levels: one supplier, M manufacturers, N retailers, K demand markets, and R recyclers. There are different levels of vertical cooperation in this network system. The closed-loop supply chain includes a forward supply chain and a reverse supply chain. The recycler is a network node that connects the forward and reverse supply chains. In this network, suppliers are responsible for providing raw materials to manufacturers; manufacturers are responsible for producing new and remanufactured products, and sending waste materials generated during production to scrap centers; retailers are responsible for selling products produced by manufacturers to Demand markets; recyclers are responsible for recycling and reselling used products to manufacturers. Due to some uncertain factors in the closed-loop supply chain, the following assumptions are made:

1. All companies are rational and are seeking to maximize their own profits;

2. Consider only the manufacture or recycling of one product or a fully substitutable product, and there is no essential difference between a remanufactured product and a new product;
3. The closed-loop supply chain is dominated by manufacturers, and decision makers at the same level are in a non-cooperative competition state;
4. The related cost function and economic function are both continuously differentiable convex functions.

2.2. Symbol Description

For ease of reference, notations used in this paper are listed below:

- q_{im} transaction quantity between vendor i and manufacture m
- q_{mn} transaction quantity between manufacture m and retailer n
- q_{nk} transaction quantity between retailer n and demand market k
- q_{rk} transaction quantity between demand market k and recycler r
- q_{mr} transaction quantity between manufacture m and recycler r
- p_{im} product transaction price between vendor i and manufacture m
- p_{mn} product transaction price between manufacture m and retailer n
- p_{nk} product transaction price between retailer n and demand market k
- p_{rk} product transaction price between demand market k and recycler r
- p_{mr} product transaction price between manufacture m and recycler r
- p_k unit product demand price in demand market k
- p unit product disposal cost
- β_1 product conversion rate of new materials, $\beta_1 \in (0,1)$
- β_2 product conversion rate of old materials, $\beta_2 \in (0,1)$
- α recovery rate of waste products in demand markets, $\alpha \in (0,1)$
- π recycler's conversion rate of waste products into usable materials
- $\bar{\pi}$ recycler's conversion rate of waste products into unusable materials
- η_1 demand intensity factor of traditional channels
- η_2 demand competition intensity factor of network channels
- δ consumer's preference factor for online channel shopping

3. Equilibrium Model of Closed-Loop Supply Chain Super-network

In this section, we will analyze the behavioral decisions and optimal goals of each supplier, manufacturer, retailer, demand market and recycler.

3.1. Supplier Behavioral Decision and Optimal Goal

The revenue of the supplier i is $p_{im}q_{im}$, the transaction cost between the supplier i and the manufacturer m is $C_{im}(q_{im})$, and the supplier's procurement cost is $f_i(\sum_{m=1}^M q_{im})$. The supplier i pursues maximizing profit. This paper uses a standard weight function of 1. The objective function of the supplier i is:

$$\max U_i = p_{im} \sum_{m=1}^M q_{im} - f_i(\sum_{m=1}^M q_{im}) - \sum_{m=1}^M C_{im}(q_{im}) \tag{1}$$

Assuming that the functions in formula (1) are all continuously differentiable convex functions, the optimal solution of the supplier i is q_{im}^* . This formula satisfies the following variational inequality:

$$\sum_{i=1}^I \sum_{m=1}^M \left[\frac{\partial f_i(q_{im}^*)}{\partial q_{im}} + \frac{\partial C_{im}(q_{im}^*)}{\partial q_{im}} - p_{im}^* \right] \times (q_{im} - q_{im}^*) \geq 0 \quad (q_{im}, p_{im}^* \geq 0, \forall i, m) \tag{2}$$

3.2. Manufacturer Behavioral Decision and Optimal Goal

The profit obtained by manufacturer m from selling the product to retailer n is $p_{mn}q_{mn}$, and the profit obtained by manufacturer m through the network channel is $p_{mk}q_{mk}$. The transaction costs of manufacturer m and supplier i , retailer n , demand market k , and recycler r are respectively $\bar{C}_{im}(q_{im}), C_{mn}(q_{mn}), C_{mk}(q_{mk}), \bar{C}_{mr}(q_{mr})$. Manufacturer's new and old materials production costs are $f_m(\beta_1, \sum_{i=1}^I q_{im}), f_m(\beta_2, \sum_{r=1}^R q_{mr})$. Same as method 3.1, the objective function of manufacturer m is:

$$\begin{aligned} \max U_m = & p_{mn} \sum_{n=1}^N q_{mn} - f_m(\beta_1, \sum_{i=1}^I q_{im}) - f_m(\beta_2, \sum_{r=1}^R q_{mr}) - \sum_{n=1}^N C_{mn}(q_{mn}) - p_{im} \sum_{i=1}^I q_{im} \\ & p_{mk} \sum_{k=1}^K q_{mk} - \sum_{k=1}^K C_{mk}(q_{mk}) - C_m(\sum_{k=1}^K q_{mk}) - \sum_{i=1}^I \bar{C}_{im}(q_{im}) - p_{mr} \sum_{r=1}^R q_{mr} - \sum_{r=1}^R \bar{C}_{mr}(q_{mr}) \\ & - p[(1-\beta_1) \sum_{i=1}^I q_{im} + (1-\beta_2) \sum_{r=1}^R q_{mr}] \end{aligned} \tag{3}$$

$$s.t. \quad \sum_{n=1}^N q_{mn} + \sum_{k=1}^K q_{mk} \leq \beta_1 \sum_{i=1}^I q_{im} + \beta_2 \sum_{r=1}^R q_{mr}$$

Assuming that the functions in equation (3) are all continuously differentiable convex functions, the optimal solution of the manufacturer m is $(q_{im}^*, q_{mn}^*, q_{mk}^*, q_{mr}^*, \lambda_1^*)$. Then the formula satisfies the following variational inequality:

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial C_{mn}(q_{mn}^*)}{\partial q_{mn}} - p_{mn}^* + \lambda_1^* \right] \times (q_{mn} - q_{mn}^*) + \sum_{m=1}^M \sum_{k=1}^K \left[\frac{\partial C_{mk}(q_{mk}^*)}{\partial q_{mk}} + \right. \\ & \left. \frac{\partial C_m(\sum_{k=1}^K q_{mk}^*)}{\partial q_{mk}} - p_{mk}^* + \lambda_1^* \right] \times (q_{mk} - q_{mk}^*) + \sum_{i=1}^I \sum_{m=1}^M \left[-\frac{\partial f_m(\beta_1, \sum_{i=1}^I q_{im}^*)}{\partial q_{im}} \right. \\ & \left. + \frac{\partial \bar{C}_{im}(q_{im}^*)}{\partial q_{im}} + p_{im}^* + p(1-\beta_1) - \beta_1 \lambda_1^* \right] \times (q_{im} - q_{im}^*) + \sum_{m=1}^M \sum_{r=1}^R \left[-\frac{\partial f_m(\beta_2, \sum_{r=1}^R q_{mr}^*)}{\partial q_{mr}} \right. \\ & \left. + \frac{\partial \bar{C}_{mr}(q_{mr}^*)}{\partial q_{mr}} + p_{mr}^* + p(1-\beta_2) - \beta_2 \lambda_1^* \right] \times (q_{mr} - q_{mr}^*) + \sum_{m=1}^M \left(\beta_1 \sum_{i=1}^I q_{im}^* + \beta_2 \sum_{r=1}^R q_{mr}^* \right. \\ & \left. - \sum_{n=1}^N q_{mn}^* - \sum_{k=1}^K q_{mk}^* \right) \times (\lambda_1 - \lambda_1^*) \geq 0 \end{aligned} \tag{4}$$

$$(q_{im}, q_{mn}, q_{mr}, q_{mk}, \lambda_1, p_{im}^*, p_{mn}^*, p_{mk}^*, p_{mr}^* \geq 0, \forall i, m, n, k, r)$$

λ_1 is the Lagrange coefficient that guarantees that the variational inequality (4) holds.

3.3. Retailer Behavioral Decision and Optimal Goal

Retailer n earns $p_{nk}q_{nk}$, The transaction costs of retailer n with manufacturer m and demand market k are $\bar{C}_{mn}(q_{mn}), C_{nk}(q_{nk})$. Retailer's product storage fee is $C_n(\sum_{m=1}^M q_{mn})$. Same as 3.1 method, the objective function of retailer n is:

$$\max U_n = p_{nk} \sum_{k=1}^K q_{nk} - p_{mn} \sum_{m=1}^M q_{mn} - \sum_{k=1}^K C_{nk}(q_{nk}) - \sum_{m=1}^M \bar{C}_{mn}(q_{mn}) - C_n(\sum_{m=1}^M q_{mn}) \tag{5}$$

$$s.t. \sum_{k=1}^K q_{nk} \leq \sum_{m=1}^M q_{mn}$$

Assuming that the functions in equation (5) are all continuously differentiable convex functions, the optimal solution for retailer n is $(q_{mn}^*, q_{nk}^*, \lambda_2^*)$. This equation satisfies the following variational inequality:

$$\begin{aligned} & \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial C_{nk}(q_{nk}^*)}{\partial q_{nk}} - p_{nk}^* + \lambda_2^* \right] \times (q_{nk} - q_{nk}^*) + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial \bar{C}_{mn}(q_{mn}^*)}{\partial q_{mn}} + \frac{\partial C_n(\sum_{m=1}^M q_{mn}^*)}{\partial q_{mn}} \right. \\ & \left. + p_{mn}^* - \lambda_2^* \right] \times (q_{mn} - q_{mn}^*) + \sum_{n=1}^N \left(\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K q_{nk}^* \right) \times (\lambda_2 - \lambda_2^*) \geq 0 \end{aligned} \tag{6}$$

$$(q_{mn}, q_{nk}, \lambda_2, p_{mn}^*, p_{nk}^* \geq 0, \forall m, n, k)$$

λ_2 is the Lagrange coefficient that guarantees that the variational inequality (6) holds.

3.4. Demand Market Equilibrium Conditions

Products in demand market k are provided by traditional channels and online channels. The demand for traditional channels is $D_k(\eta_1, p_k^*, \delta)$, and the demand of the network channel is $D_k(\eta_1, p_k^{e*}, \delta)$. Then the equilibrium condition of the demand market k is:

$$D_k(\eta_1, p_k^*, \delta) \begin{cases} = \sum_{n=1}^N q_{nk}^*, & \text{if } p_k^* > 0 \\ \leq \sum_{n=1}^N q_{nk}^*, & \text{if } p_k^* = 0 \end{cases} \tag{7}$$

$$p_{nk}^* + \bar{C}_{nk} \begin{cases} = p_k^*, & \text{if } q_{nk}^* > 0 \\ \geq p_k^*, & \text{if } q_{nk}^* = 0 \end{cases} \tag{8}$$

$$D_k(\eta_1, p_k^{e*}, \delta) \begin{cases} = \sum_{m=1}^M q_{mk}^*, & \text{if } p_k^{e*} > 0 \\ \geq \sum_{m=1}^M q_{mk}^*, & \text{if } p_k^{e*} = 0 \end{cases} \tag{9}$$

$$p_{mk}^* + \bar{C}_{mk} \begin{cases} = p_k^{e*}, & \text{if } q_{mk}^* > 0 \\ \geq p_k^{e*}, & \text{if } q_{mk}^* = 0 \end{cases} \tag{10}$$

Formulas (7) and (9) respectively indicate that under equilibrium conditions, if consumers in the demand market are willing to purchase such products through traditional channels and online channels, then the demand is equal to the supply; when the supply is greater than the demand, the demand price of such products is zero. Formulas (8) and (10) respectively indicate that when the demand market and retailer, the demand market and manufacturer's unit product transaction price and unit transaction cost do not exceed the unit demand price of the product, the demand market and the retailer, manufacturer conduct transactions; otherwise, no transaction occurs.

$$\alpha_k \left(\sum_{k=1}^K q_{rk}^* \right) \begin{cases} = p_{rk}^*, & \text{if } q_{rk}^* > 0 \\ \geq p_{rk}^*, & \text{if } q_{rk}^* = 0 \end{cases} \quad (11)$$

$$s.t. \sum_{r=1}^R q_{rk} \leq \alpha \left(\sum_{n=1}^N q_{nk} + \sum_{m=1}^M q_{mk} \right)$$

The negative utility function in Equation (11) $\alpha_k \left(\sum_{r=1}^R q_{rk}^* \right)$ indicating that consumers are willing to return used products [9]. It is an increasing function of the recycler's recovery amount, that is, the more waste products returned by consumers, the more trouble they bring to themselves, so the consumer hopes that the recycler can give more compensation and give a higher recycling price. Equation (11) states that when the unit transaction cost between the demand market and the recycler does not exceed the unit transaction price of the product, the demand market conducts transactions with the recycler; otherwise, no transaction occurs. Constraints indicate that the total amount of products recycled by the recycler to the demand market cannot exceed the supply in the demand market. α is the product recovery rate.

Assuming that the functions in (7), (8), (9), (10), and (11) are all continuously differentiable convex functions, the optimal solution of the demand market k is $(q_{nk}^*, q_{mk}^*, q_{rk}^*, \lambda_3^*)$, then the formula satisfies the following variational inequality:

$$\begin{aligned} & \sum_{k=1}^K \left[\sum_{n=1}^N q_{nk}^* - D_k(\eta_1, p_k^*, \delta) \right] \times (p_k - p_k^*) + \sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial \bar{C}_{nk}(q_{nk}^*)}{\partial q_{nk}} + p_{nk}^* - p_k^* - \alpha \lambda_3^* \right] \\ & \times (q_{nk} - q_{nk}^*) + \sum_{k=1}^K \left[\sum_{m=1}^M q_{mk}^* - D_k(\eta_2, p_k^{e*}, \delta) \right] \times (p_k^e - p_k^{e*}) + \sum_{m=1}^M \sum_{k=1}^K \left[\frac{\partial \bar{C}_{mk}(q_{mk}^*)}{\partial q_{mk}} + \right. \\ & \left. p_{mk}^* - p_k^{e*} - \alpha \lambda_3^* \right] \times (q_{mk} - q_{mk}^*) + \sum_{r=1}^R \sum_{k=1}^K \left[\alpha_k \left(\sum_{r=1}^R q_{rk}^* \right) - p_{rk}^* + \lambda_3^* \right] \times (q_{rk} - q_{rk}^*) + \\ & \sum_{k=1}^K \left(\alpha \sum_{n=1}^N q_{nk}^* + \alpha \sum_{m=1}^M q_{mk}^* - \sum_{r=1}^R q_{rk}^* \right) \times (\lambda_3 - \lambda_3^*) \geq 0 \end{aligned} \quad (12)$$

$$(q_{nk}, q_{mk}, q_{rk}, \lambda_3, p_{nk}^*, p_{mk}^*, p_{rk}^*, p_k^* \geq 0, \forall m, n, k, r)$$

λ_3 is the Lagrange coefficient that guarantees that the variational inequality (12) holds.

3.5. Recycler Behavioral Decision and Optimal Goal

Recycler earns $p_{mr} q_{mr}$, the transaction costs between recycler r and demand market k , and manufacturer m are $\bar{C}_{rk}(q_{rk}), C_{mr}(q_{mr})$. Recycler's acquisition, transportation, and storage costs are $C_r \left(\sum_{k=1}^K q_{rk} \right)$ [10]. Same as the method 3.1, the objective function of the recycler r is:

$$\max U_r = p_{mr} \sum_{m=1}^M q_{mr} - \sum_{m=1}^M C_{mr}(q_{mr}) - p_{rk} \sum_{k=1}^K q_{rk} - \sum_{k=1}^K \bar{C}_{rk}(q_{rk}) - C_r \left(\sum_{k=1}^K q_{rk} \right) - p_{\pi} \sum_{k=1}^K q_{rk} \quad (13)$$

$$s.t. \sum_{m=1}^M q_{mr} \leq \pi \sum_{k=1}^K q_{rk}$$

Assuming that the functions in equation (13) are all continuously differentiable convex functions, the optimal solution for the recycler r is $(q_{mr}^*, q_{rk}^*, \lambda_4^*)$, This formula satisfies the following variational inequality:

$$\begin{aligned} & \sum_{m=1}^M \sum_{r=1}^R \left[\frac{\partial C_{mr}(q_{mr}^*)}{\partial q_{mr}} - p_{mr}^* + \lambda_4^* \right] \times (q_{mr} - q_{mr}^*) + \sum_{k=1}^K \sum_{r=1}^R \left[\frac{\partial \bar{C}_{rk}(q_{rk}^*)}{\partial q_{rk}} + \frac{\partial C_r(\sum_{k=1}^K q_{rk}^*)}{\partial q_{rk}} \right. \\ & \left. + p_{rk}^* + p\bar{\pi} - \pi\lambda_4^* \right] \times (q_{rk} - q_{rk}^*) + \sum_{r=1}^R \left(\pi \sum_{k=1}^K q_{rk}^* - \sum_{m=1}^M q_{mr}^* \right) \times (\lambda_4 - \lambda_4^*) \geq 0 \\ & (q_{mr}, q_{rk}, \lambda_4, p_{mr}^*, p_{rk}^* \geq 0, \forall k, r, m) \end{aligned} \tag{14}$$

λ_4 is the Lagrange coefficient that guarantees that the variational inequality (14) holds.

3.6. Model Dynamic Equilibrium Condition Analysis

According to the above formula, the only solution for the dynamic equilibrium state of the closed-loop supply chain super-network model is:

$$(q_{im}^*, q_{mn}^*, q_{nk}^*, q_{mk}^*, q_{rk}^*, q_{mr}^*, p_k^*, p_k^{e*}, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^* \geq 0, \forall i, m, n, k, r),$$

and meet the following conditions:

$$\begin{aligned} & \sum_{m=1}^M \sum_{i=1}^I \left[\frac{\partial f_i(\sum_{m=1}^M q_{im}^*)}{\partial q_{im}} + \frac{\partial f_m(\beta_1, \sum_{i=1}^I q_{im}^*)}{\partial q_{im}} + \frac{\partial C_{im}(q_{im}^*)}{\partial q_{im}} + \frac{\partial \bar{C}_{im}(q_{im}^*)}{\partial q_{im}} + p(1 - \beta_1) - \beta_1 \lambda_1^* \right] \\ & \times (q_{im} - q_{im}^*) + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial C_{mn}(q_{mn}^*)}{\partial q_{mn}} + \frac{\partial \bar{C}_{mn}(q_{mn}^*)}{\partial q_{mn}} + \frac{\partial C_n(\sum_{m=1}^M q_{mn}^*)}{\partial q_{mn}} + \lambda_1^* - \lambda_2^* \right] \times (q_{mn} - q_{mn}^*) \\ & + \sum_{m=1}^M \sum_{k=1}^K \left[\frac{\partial C_{mk}(q_{mk}^*)}{\partial q_{mk}} + \frac{\partial \bar{C}_{mk}(q_{mk}^*)}{\partial q_{mk}} + \frac{\partial C_m(\sum_{k=1}^K q_{mk}^*)}{\partial q_{mk}} - p_k^{e*} + \lambda_1^* - \alpha \lambda_3^* \right] \times (q_{mk} - q_{mk}^*) \\ & + \sum_{k=1}^K \sum_{n=1}^N \left[\frac{\partial C_{nk}(q_{nk}^*)}{\partial q_{nk}} + \frac{\partial \bar{C}_{nk}(q_{nk}^*)}{\partial q_{nk}} - p_k^* + \lambda_2^* - \alpha \lambda_3^* \right] \times (q_{nk} - q_{nk}^*) + \sum_{r=1}^R \sum_{k=1}^K \left[\alpha_k \left(\sum_{r=1}^R q_{rk}^* \right) \right. \\ & \left. \frac{\partial C_r(\sum_{k=1}^K q_{rk}^*)}{\partial q_{rk}} + \frac{\partial \bar{C}_{rk}(q_{rk}^*)}{\partial q_{rk}} + p\bar{\pi} + \lambda_3^* - \pi\lambda_4^* \right] \times (q_{rk} - q_{rk}^*) + \sum_{m=1}^M \sum_{r=1}^R \left[\frac{\partial f_m(\beta_2, \sum_{r=1}^R q_{mr}^*)}{\partial q_{mr}} + \right. \\ & \left. \frac{\partial C_{mr}(q_{mr}^*)}{\partial q_{mr}} + \frac{\partial \bar{C}_{mr}(q_{mr}^*)}{\partial q_{mr}} + p(1 - \beta_2) + \lambda_4^* - \beta_2 \lambda_1^* \right] \times (q_{mr} - q_{mr}^*) + \sum_{k=1}^K \left[\sum_{n=1}^N q_{nk}^* - D_k(\eta_1, p_k^*, \delta) \right] \\ & \times (p_k - p_k^*) + \sum_{k=1}^K \left[\sum_{m=1}^M q_{mk}^* - D_k(\eta_2, p_k^{e*}, \delta) \right] \times (p_k^e - p_k^{e*}) + \sum_{m=1}^M \left(\beta_1 \sum_{i=1}^I q_{im}^* + \beta_2 \sum_{r=1}^R q_{mr}^* - \sum_{n=1}^N q_{mn}^* \right. \\ & \left. - \sum_{k=1}^K q_{mk}^* \right) \times (\lambda_1 - \lambda_1^*) + \sum_{n=1}^N \left(\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K q_{nk}^* \right) \times (\lambda_2 - \lambda_2^*) + \sum_{k=1}^K \left(\alpha \sum_{n=1}^N q_{nk}^* + \alpha \sum_{m=1}^M q_{mk}^* - \sum_{r=1}^R q_{rk}^* \right) \\ & \times (\lambda_3 - \lambda_3^*) + \sum_{r=1}^R \left(\pi \sum_{k=1}^K q_{rk}^* - \sum_{m=1}^M q_{mr}^* \right) \times (\lambda_4 - \lambda_4^*) \geq 0 \end{aligned} \tag{15}$$

Proof: Adding the above formulas (2), (4), (6), (12), and (14) to simply obtain formula (15). If we add $-p_{im}^* + p_{im}^*$ to the first bracket, add $-p_{mn}^* + p_{mn}^*$ to the second bracket, add $-p_{mk}^* + p_{mk}^*$ to the third bracket, add $-p_{nk}^* + p_{nk}^*$ to the fourth bracket, add $-p_{rk}^* + p_{rk}^*$ to the fifth bracket, and add $-p_{mr}^* + p_{mr}^*$ to the sixth bracket in formula (15), it will not affect the solution to formula (15). Therefore, the solution of formula (15) is the sum of formula (2), (4), (6), (12), and (14). Finally, the gradient projection algorithm is used to solve the variational inequality of the continuous convex function.

4. Numerical Examples

Assume that the network consists of two suppliers, two manufacturers, two retailers, two demand markets and two recyclers. Assume that $\beta_1=0.4, \beta_2=0.6, \alpha=0.8, p=2, \pi=0.4, \bar{\pi}=0.6$. The purchasing cost of suppliers is given by:

$$f_i(\sum_{m=1}^2 q_{im}) = (\sum_{m=1}^2 q_{im})^2 + 0.5(\sum_{m=1}^2 q_{im})(\sum_{m=1}^2 q_{im}) + 2\sum_{m=1}^2 q_{im} \quad i=1,2$$

The manufacturers' production cost of new material is given by:

$$f_m(0.4, \sum_{i=1}^2 q_{im}) = 1.5[0.4(\sum_{i=1}^2 q_{im})^2] + 2(\sum_{i=1}^2 q_{im})(\sum_{i=1}^2 q_{im}) + 5 \quad m=1,2$$

The manufacturers' production cost of old material is given by:

$$f_m(0.6, \sum_{r=1}^2 q_{mr}) = 0.5[0.6(\sum_{r=1}^2 q_{mr})^2] + 0.6(\sum_{r=1}^2 q_{mr}) + 2 \quad m=1,2$$

The production storage cost of retailers is given by:

$$C_n(\sum_{m=1}^2 q_{mn}) = 0.5(\sum_{m=1}^2 q_{mn})^2 \quad n=1,2$$

The product acquisition, shipping and storage costs of recyclers is given by:

$$C_r(\sum_{k=1}^2 q_{rk}) = (\sum_{k=1}^2 q_{rk})^2 + 3(\sum_{k=1}^2 q_{rk})(\sum_{k=1}^2 q_{rk}) + 2 \quad m, r=1,2$$

The transaction cost between suppliers and manufacturers, the transaction cost between manufacturers and retailers, the transaction cost between retailer and demand markets, the transaction between demand markets and recyclers, the transaction cost between manufacturers and recyclers are given by:

$$C_{im}(q_{im}) = 0.5q_{im}^2 + q_{im} \quad C_{mn}(q_{mn}) = 0.5q_{mn}^2 \quad C_{mk}(q_{mk}) = 0.5q_{mk}^2 + 0.5q_{mk}$$

$$C_{rk}(q_{rk}) = 0.5q_{rk}^2 + 0.5q_{rk} \quad C_{mr}(q_{mr}) = 0.5q_{mr}^2 + 0.5q_{mr} \quad i, m, n, k, r=1,2$$

The cost of manufacturers operating online channels is given by:

$$C_m(\sum_{k=1}^2 q_{mk}) = \sum_{k=1}^2 q_{mk} \quad m=1,2$$

Negative utility function of consumers willing to sell used products to recyclers is given by:

$$\alpha_k(\sum_{r=1}^2 q_{rk}) = 0.5(\sum_{k=1}^2 \sum_{r=1}^2 q_{rk}) + 5 \quad k=1,2$$

The product demand function of traditional channels is given by:

$$D_k(\eta_1, P_k, \delta) = -1.5p_k - 2p_{3-k} + \eta_1(p_k^e + p_{3-k}^e) + 825(1 - \delta)$$

The product demand function of network channels is given by:

$$D_k(\eta_2, p_k^e, \delta) = -0.5p_k^e - 0.8p_{3-k}^e + \eta_2(p_k + p_{3-k}) + 800\delta$$

4.1. Equilibrium Results under the Influence of Traditional Channel Competition Intensity Factors

Under these perceived production cost function and economic function, using MatlabR2016a software for programming and simulation experiments and setting the step size of the iteration to 0.01 and the calculation accuracy to 0.001. The test results of the equilibrium model of closed-loop supply chain super-network are summarized in Table 1. And U means profit.

Table 1: Closed-loop supply chain super-network equilibrium results

variable	$\eta_1 = 0$	$\eta_1 = 0.02$	$\eta_1 = 0.04$	$\eta_1 = 0.06$	$\eta_1 = 0.08$	$\eta_1 = 0.1$
$q_{im}(i, m = 1, 2)$	5.5569	5.5902	5.6237	5.6574	5.6913	5.7253
$q_{mn}(m, n = 1, 2)$	0.7980	0.9948	1.1927	1.3921	1.5928	1.7948
$q_{mk}(m, k = 1, 2)$	2.5757	2.3961	2.2148	2.0328	1.8496	1.6653
$q_{nk}(n, k = 1, 2)$	1.0225	1.2198	1.4180	1.6179	1.8191	2.0216
$q_{rk}(r, k = 1, 2)$	3.0870	3.1013	3.1147	3.1291	3.1436	3.1583
$q_{mr}(m, r = 1, 2)$	1.4963	1.5023	1.5075	1.5153	1.5195	1.5256
$p_k(k = 1, 2)$	176.1935	178.1090	180.0372	181.9776	183.9307	185.8965
$p_k^e(k = 1, 2)$	176.9254	177.4962	178.0716	178.6498	179.2318	179.8176
$U_i(i = 1, 2)$	41.3732	51.5106	61.7598	72.1255	82.5965	93.1917
$U_m(m = 1, 2)$	70.7453	75.8240	77.1626	80.7659	81.3095	85.9306
$U_n(n = 1, 2)$	59.9759	61.7824	63.2392	64.4789	65.3799	65.9571
$U_r(r = 1, 2)$	4.3410	7.6843	11.0433	12.5438	13.5813	14.4799

Table 1 shows that with the increase of the traditional channel competition intensity factor, the transaction volume on traditional channels has increased, that is, the transaction volume between manufacturers and retailers, retailers and demand markets has increased. And the transaction volume of network channels has decreased, that is, the transaction volume of manufacturers and demand markets has decreased. This is because as the competition intensity factor of traditional channels increases, the competitiveness of traditional channels increases, and the products sold through traditional channels increase. In contrast, the competitiveness

of network channels weakens. In addition, with the increase in the intensity of traditional channel competition factors, the profits of suppliers, manufacturers, retailers and recyclers have increased, which indicates that the enhanced competitiveness of traditional channels has positive feedback on all layers of companies in the closed-loop supply chain effect.

4.2. Equilibrium Results under the Influence of Network Channel Competition Intensity Factors

Same as 4.1, carry out simulation experiments. The test results of the equilibrium model of closed-loop supply chain super-network are summarized in Table 2.

Table 2: Closed-loop supply chain super-network equilibrium results

variable	$\eta_2 = 0$	$\eta_2 = 0.02$	$\eta_2 = 0.04$	$\eta_2 = 0.06$	$\eta_2 = 0.08$	$\eta_2 = 0.1$
$q_{im}(i, m = 1, 2)$	5.2797	5.2902	5.3086	5.4748	5.5996	5.7253
$q_{mn}(m, n = 1, 2)$	3.5637	3.5688	3.5538	2.9538	2.3763	1.7948
$q_{mk}(m, k = 1, 2)$	0	0	0	0.3803	1.0208	1.6653
$q_{nk}(n, k = 1, 2)$	3.8873	3.8924	3.8672	3.1799	2.6028	2.0216
$q_{rk}(r, k = 1, 2)$	3.3620	3.3660	3.3357	3.0567	3.1076	3.1583
$q_{mr}(m, r = 1, 2)$	1.7114	1.7129	1.6953	1.4843	1.5051	1.5256
$p_k(k = 1, 2)$	183.3393	183.6592	183.9979	184.6877	185.2900	185.8965
$p_k^e(k = 1, 2)$	153.8462	159.4958	165.1535	170.2445	175.0152	179.8176
$U_i(i = 1, 2)$	49.4546	59.7483	69.8965	76.7277	80.7160	93.1917
$U_m(m = 1, 2)$	29.1842	44.3038	56.7022	68.8965	76.7945	85.9306
$U_n(n = 1, 2)$	107.3242	93.7652	85.3935	76.4592	70.4580	65.9571
$U_r(r = 1, 2)$	1.4328	3.7607	7.1035	12.3131	13.6072	14.4799

Table 2 shows that the competition intensity factor of network channels increases, the transaction volume on network channels increases, that is, the transaction volume between manufacturers and the demand market increases; while the transaction volume on traditional channels decreases, that is, the volume of transactions between manufacturers and retailers, retailers and demand markets has decreased. Just as during special festivals such as the Jingdong 618 Shopping Festival and the Taobao Double 11 Carnival Shopping Festival, the

online channels will carry out large-scale promotional activities to stimulate consumers' shopping needs. Consumers are more willing to buy cost-effective equivalent products than to go to physical stores to buy such products. In addition, as the competition intensity factor for online channels increases, the profits of suppliers, manufacturers and recyclers increase, while the profits of retailers decrease. This shows that the increased competitiveness of online channels has caused a certain impact on traditional channels, resulting in damage to the interests of retailers and detriment to the operation of retailers.

4.3. Equilibrium Results under the Influence of Consumer Channel Preferences

Same as 4.1, carry out simulation experiments. Assume $\eta_1 = 0.1, \eta_2 = 0.1, \delta = 0.2 \sim 0.25$. The test results of the equilibrium model of closed-loop supply chain super-network are summarized in Table 3 and Figure 1 to Figure 4.

Table 3: Closed-loop supply chain super-network equilibrium results

variable	$\alpha=0.2$	$\alpha=0.21$	$\alpha=0.22$	$\alpha=0.23$	$\alpha=0.24$	$\alpha=0.25$
$q_{im}(i, m = 1, 2)$	5.6621	5.5963	5.5301	5.5056	5.6289	5.7253
$q_{mn}(m, n = 1, 2)$	3.7523	3.7207	3.6887	3.5404	2.6551	1.7948
$q_{mk}(m, k = 1, 2)$	0	0	0	0	0.7573	1.6653
$q_{nk}(n, k = 1, 2)$	4.0775	4.0446	4.0128	3.8194	2.8819	2.0216
$q_{rk}(r, k = 1, 2)$	3.5142	3.4878	3.4625	3.2758	3.1202	3.1583
$q_{mr}(m, r = 1, 2)$	1.7724	1.7618	1.7519	1.6323	1.5104	1.5256
$p_k(k = 1, 2)$	194.9715	192.9672	190.9623	189.0534	187.5044	185.8965
$p_k^e(k = 1, 2)$	153.0652	158.9107	164.7561	170.5961	175.3080	179.8176

As can be seen from Table 1 above, with the increase of consumers' preference factors for online shopping, the transaction volume of online channels has increased, while the transaction volume of traditional channels has decreased. This is because with the development of the Internet and the progress of the times, more and more consumers choose online shopping, and online shopping can be completed online without the consumer going out, and delivery is more convenient. Therefore, some consumers prefer online shopping. When the online channel preference factor increases, the demand price of products sold through the online channel increases, while the demand price of products sold through the traditional channel decreases. This shows that the preference factor of online channels is positively related to changes in the demand price of online channel products, and negatively related to changes in the demand price of traditional channel products.

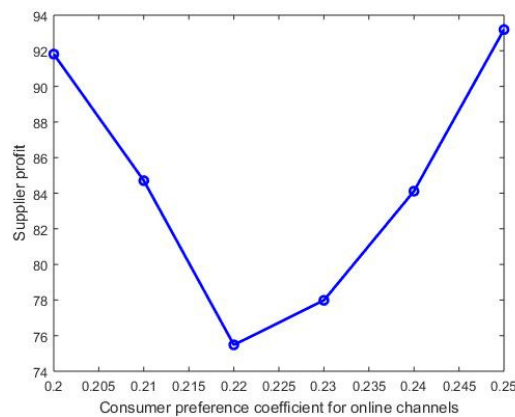


Figure 1: Impact of channel preference factors on supplier profits

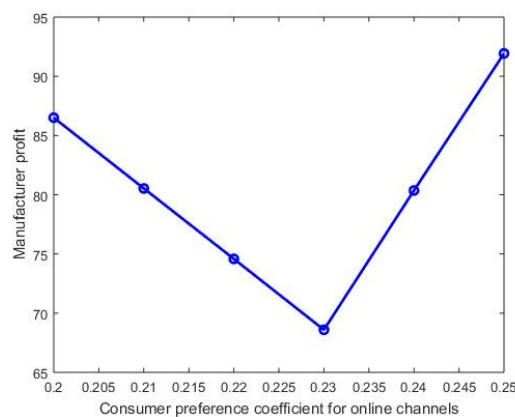


Figure 2: Impact of channel preference factors on manufacturer profits

As can be seen from Figure 1 above, as the network channel preference factor increases, the supplier's profit decreases first and then increases. This is because the increase of the network channel preference factor causes the transaction volume between the supplier and the manufacturer to decrease first and then increase, and changes in trading volume affect profits. In Figure 2 above, when the online channel preference factor increases, the manufacturer's profit decreases first and then increases, because in the 0.2-0.23 range, the transaction volume between the manufacturer and the retailer continues to decrease, while the transaction between the manufacturer and the demand market. If the quantity is 0, so the profit of the manufacturer is also decreasing. In the range of 0.23-0.25, although the transaction volume between manufacturers and retailers continues to decrease, the transaction volume between manufacturers and demand markets has increased rapidly. The profit gained from products sold through online channels makes up for the loss of profits from traditional channels. Eventually, manufacturers' profits have increased.

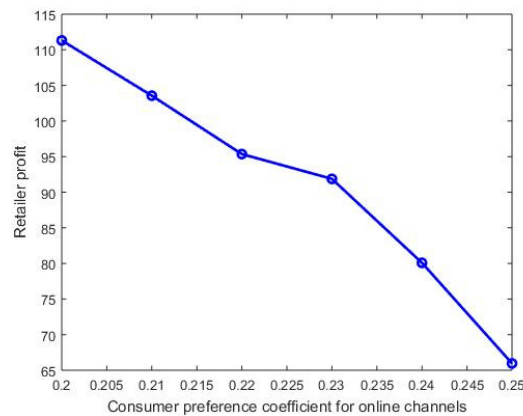


Figure 3: Impact of channel preference factors on retailer profits

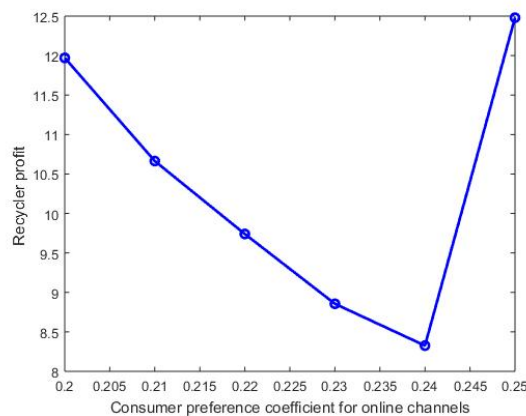


Figure 4: Impact of channel preference factors on recycler profits

As can be seen from Figure 3, the retailer's profit decreases as the online channel preference factor increases. When the online channel preference factor increases, the transaction volume between manufacturers and retailers, retailers and demand markets continue to decrease. At this time, more and more consumers use online channels for shopping or some consumers' preferences for online shopping are deepened, resulting in a decrease in retailer sales and therefore a decrease in retailer profits. It can be seen from Figure 4 that the profit of recyclers increases with the increase of network channel preference factors, indicating a positive correlation between the two.

5. Conclusion

In the case where the manufacturer opens up the network channel and forms a dual channel, considering the two factors of the consumer's preference for the network channel and the competition between the traditional channel and the network channel, this paper establishes a closed-loop supply chain super-network model. And this paper studies the behavioral decision-making and optimal goals of enterprises in each layer of the closed-loop supply chain network. Finally, the equilibrium point of the system network is obtained by using variational inequality theory and gradient projection algorithm. Through numerical example analysis, the following conclusions are obtained:

1. When the traditional channel competition intensity factor increases, changes in the transaction volume of traditional channels in the closed-loop supply chain, the profits of various layers of companies, and the demand and price of the two sales channels are positively related

to the competition intensity factor between channels; while the change in the transaction volume of online channels competition intensity factor between channels is negatively correlated.

2. When the competition intensity factor of network channels increases, the competition intensity factor is positively related to changes in transaction volume, demand price, and corporate profits related to network channels, and negatively related to transaction volume, demand price, and corporate profits related to traditional channels.

3. As consumers' preference for online channels increases, the profits of suppliers and manufacturers first decrease and then increase, the profits of retailers decrease, and the profits of recyclers increase. This shows that the increase in the preference of online channels is very detrimental to the operation of retailers. On the one hand, it is recommended that retailers strengthen their own operational capabilities to attract consumers, such as providing personalized services and improving the quality of after-sales services, so that consumers are loyal to using traditional channels for shopping; on the other hand, manufacturers should coordinate the competitiveness of the two channels, and try to avoid the negative impact of online channels on traditional channels.

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