

A Test of Violations and Trading Strategies in China's CSI 300 Index Options Market

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Abstract

In this paper we study and test boundary violation, convexity violation, put-call parity violation, mispricing from Black-Scholes model and relevant trading strategies in China's CSI 300 Index Options market from 2015 to 2020. We find that there is no upper bound violation. Lower bound violation frequency decreases as moneyness decreases and is higher for put options than call options. The convexity violation ratio decreases while the profitability increases after transaction costs are taken into consideration. Violation frequency of the put-call parity principle is surprisingly high when compared to other violations, and a significance profit can be gained by taking advantage of the mispricing. In contrast, the delta hedge strategy is subject to a significant loss even adopting different trading signals.

Keywords

Option mispricing, Boundary violation, Convexity violation, Put-call parity, Delta hedge.

1. Introduction

Option is a contract between two parties which provides the right, but not the obligation, to one party to sell or buy an asset, under certain conditions, within a specified period of time[1]. Options are becoming increasingly important in global derivatives market, particularly in the function of risk management. Some theories of options' pricing such as rules of boundary conditions have been established to help price options properly and assist investors to adjust trading strategies. However, in the real world, mispricing is still available from which investors usually can gain huge profits. This paper will focus on Chinese CSI300 Index Options market, examining boundary violations, convexity violations, violations of put-call parity principle and Black-Scholes Model, identifying and analyzing arbitrage opportunities and their profitability.

In previous studies, the violation of lower bound conditions in CNX Nifty Index Options market has been tested and results indicate that at-the-money options are less likely to be mispriced [2]. There are fewer violations of PCP relation for at-the-money options, when investigating options on futures on the Standard and Poor's 500[3]. There is a high frequency of upper bound violation for US index call options[4]. In terms of option trading in Chinese market, "the price discovery in price disagreement between the China ETF 50 index and option markets using 1 year of data" has been studied[5]. Research finds a statistically

significant mispricing in Baosteel's call option over five-month sample period by adopting Black-Scholes model[6].

Our paper improves the limitations of previous studies from multiple aspects. First, instead of investigating a single type of violation, we test four types of violations: Boundary condition violation, convexity violation, put-call parity violation and Black-Scholes model violation. Second, our sampling period is extended from previous several months to five years. A larger sample base generates more accurate and convincing results. Third, we investigate all calls and puts in CSI300 index options market, within sample period, rather than only focusing on Baosteel's call option as mentioned above. This helps to generalize conclusions. Fourth, our paper takes margin, percentage of available rate of return, dividend and many other trading factors into consideration, leading to more convincing and practical results. Last, our study narrows the research gap because only a few studies have been conducted towards the emerging CSI300 index options market of which the frequency and pattern of violations are significantly different from other markets. Hence, our study does present fresh results and unique characteristics of CSI300 index option market.

The rest of the paper will present literature reviews, introduce the data and methodologies, discuss and conclude test findings and empirical results.

2. Literature Review

It is crucial to understand the nature of option returns because options returns are associated with options risk. Leverage and curvature of option payoffs are two separate components that compose the option risks. Coval and Shumway[7] test the option beta and find that there is a risk premium for options so they should be nonredundant assets. Unlike many researchers who mainly study volatiles or implied distributions, it is the first paper researching both theoretical and empirical nature of option returns. Considering options levels of systematic risk, the authors find considerable evidence that both call and put contracts earn exceedingly low returns. Also, their results strongly suggest that other factors other than market risk can also be essential to price the risk associated with option contracts. However, what is "something" here is not explained in this paper and it is surveyed by later academics.

The relationship between stock momentum and index option prices has been tested by Amin, Coval and Seyhun[8]. In the context of Black-Scholes model, there is a gap between theory assumptions and realistic option pricing. In order to explain the deviation of option values from their fair values under real-world market conditions, they test American option boundary violation and regress the bound values on 10- to 100- day past stock returns. Because of volatility surface, they control option moneyness and time to maturities when doing regression. They conclude that When past stock returns are positive, there is pressure putting up call option prices. Similarly, when past stock returns are negative, there is pressure putting up put option prices. They also find that past stock returns significantly impact the accuracy of previous option pricing models, which results in volatility smiles. They also suggest that considering the factor of past stock returns in option pricing may be more appropriate. This is an unprecedented and essential contribution because it empirically presents the influence of past stock market momentum on option prices.

Later, other researchers test this effect in different derivative markets. For example, price pressure, based on the research of the SSE 50 ETF option in the Chinese derivatives market, can be influenced by the momentum effects in the underlying stock market[8]. These series of researches partly explain the phenomenon of the existing of mispricing in options from the perspective of stock momentum.

Deviations between historical realized volatility and implied volatility estimates are a signal of volatility mispricing [9]. Two strategies of straddles portfolios and delta-hedging are

employed to discover arbitrage opportunities[9]. The evidence presented in this paper suggests that the information contained in historical realized volatility and implied volatility allows one to construct profitable trading strategies. Although option pricing is beyond the scope of this paper, the authors make some progress towards identifying an alternative estimate of implied volatility.

Delta-hedged equity option return is negatively related to the idiosyncratic stock volatility in individual U.S. stock and stock option[10]. The total volatility is decomposed to systematic volatility and idiosyncratic volatility. ($VOL^2 = SysVOL^2 + IVOL^2$). The authors apply Fama-MacBeth regression meanwhile controlling a series of factors such as volatility risk and jump risk. However, Tunic and West[11] argue that the cross-section option returns and idiosyncratic volatility of underlying asset have a positive relationship, while Fama & Macbeth[12] and Novy-Marx[13] state that they are not significantly relevant. Until now, the idiosyncratic volatility puzzle is still a mystery to academic researchers. This is a meaningful discussion to explore the option mispricing from the analysis of idiosyncratic volatility of stocks. Like other researchers, we adopt the methodology of Cao and Han[10] to calculate the delta-hedged gain.

Delta-hedged option returns are negatively associated with volatility of volatility (VOV)[14]. Three measures of volatility – implied volatility, EGARCH volatility from daily returns, realized volatility from high-frequency data – were used and the result holds in these three measures. Their findings suggest that single-name options with high uncertainty are usually be charged at a high premium because it is more difficult to hedge for these stock options. Since we can hardly predict future stock prices, in our paper we use delta-hedging strategy to exploit option mispricing. Previous studies find that a large deviation between implied volatility and realized volatility[9], high idiosyncratic volatility[10] and volatility term structure[15] are related to lower delta-hedged equity option returns. This paper documents a robust negative relationship between volatility-of-volatility and future delta-hedged option returns.

3. Data and Methodology

We obtained historical data for the CSI 300 Index Options and the Chinese stock index from the Chinese Stock Market Accounting Research (CSMAR) database. The CSMAR database is developed by Guotaian Limited Corporation, providing comprehensive and high-quality economic and financial information for academic research, especially in the Chinese market. The notations for the variables used to describe the options are summarized in table 1. The average daily trading volume of put options and call options is 1,304 contracts and 1,406 contracts respectively from 9 February 2015 to 7 February 2020. Data points with less than 1 contract were deemed illiquid and removed from the sample. After data cleaning, there are 36,922 paired observations for put and call options. Put options and call options are matched in pairs based on the trading date and symbol, with the same exercise price and expiration date.

In the actual trading process, market impact cost, including transaction costs, margins and many other factors must be included. This impacts the arbitrage rate of return. In the following section, we estimate the total market impact cost according to the respective transaction cost variables and eventually calculate it as a certain proportion of the price. According to China's supervision policy, no tax is imposed on individual investors who gain profits from buying and selling stocks and options in the Chinese market. Therefore, the tax factor is not considered.

Table 1. Summary of variable notation

Variables	Descriptions
T	Days to maturity/365
P	Put option price
C	Call option price
S	Close price of CSI 300 index
X	Exercise price of CSI 300 index
r_f	Annualized risk-free interest rate for borrowing funds
γ	Continuously dividend yield
S/X	Moneyness
N	The number corresponding to the underlying asset of each option
λ	Percentage of available rate of return for the CSI 300 market

In our paper, we adopt the following settings as adjustments to arbitrage strategies.

(1)Liquidity Factors: Most previous studies use bid-ask spread to represent liquidity. Zhang and Watada[16] suggest establishing the bid-ask spread as a constant value. Three types of value are used: 1%, 2% and 5%. Our paper covers data from 2015 to 2020. During this period, the average turnover of the CSI 300 index option was CNY 1.517 billion per year. The average daily turnover is estimated to be CNY 530 million. The average turnover of the CSI 300 index option was 0.137 million hands per year, average daily turnover is approximate 500 hands. Roughly speaking, there were only about 80,134 individual investors in 2015. Hence the liquidity condition for this option is not quite good, and the bid-ask spread is set to 5%.

(2)Dividend Factors: Dividend factors must be considered given that the diverse CSI 300 index includes many dividend paying stocks.

3.1. Boundary Arbitrage Condition

Past studies have shown that no matter what the market conditions are, upper bound restrictions are generally satisfied [16]. In other words, option prices are not supposed to rise beyond an upper bound as it leads to immediate arbitrage profits. However, lower bound conditions need to be tested.

Taking dividend and transaction cost situations into consideration, each call and put must satisfy the following conditions [17]:

$$C \geq \max(\lambda * (S - D) - X * e^{-r*T} - t_s, 0) - t_c$$

$$P \geq \max(X * e^{-r*T} - (S - D) - t_s, 0) - t_p$$

Lower bond conditions indicate that each option must be worth at least its intrinsic value. For calls, statistically, intrinsic value is the difference between the dividend-adjusted price of the underlying asset and the net present value of the strike price. For puts, it equals to net present value of the strike price minus the dividend adjusted price of the underlying asset.

3.2. Convexity Arbitrage Strategy

Convexity is a measure indicating a non-linear relationship between the value of option and the price of underlying assets. If the convexity condition is not satisfied, arbitrageurs can use the butterfly spread to gain risk free profit or engage in arbitrage. More specifically, the

arbitrageur will carry out this spread by buying one contract of the call with the highest strike price, and one contract of the call with the lowest strike price and selling two contracts of the call with the middle strike price. This will be profitable because of the following: suppose the convexity condition is violated, i.e. for $X_1 < X_2 < X_3$ satisfying $X_3 - X_2 = X_2 - X_1$, we have $C_1 - C_2 < C_2 - C_3$. Carrying out the above strategy will yield the arbitrageur

$$-C_1 - C_3 + 2C_2$$

Rearrange the terms in the above inequality, it will turn out to be exactly

$$0 < -C_1 - C_3 + 2C_2$$

Hence, with this strategy, the initial cashflow is positive.

Next, different cases are considered in terms of stock prices S at maturity.

If $S < X_1 < X_2 < X_3$, none of the calls will be exercised. This means that there will be no additional cashflow.

If $X_1 < S < X_2 < X_3$, then the arbitrageur can exercise the call with lowest exercise price to buy the stock at X_1 and sell it immediately at S , making an additional profit of $S - X_1$. If $X_1 < X_2 < S < X_3$, then the arbitrageur will exercise a call and two calls will be exercised against him, yielding a cashflow of $(S - X_1) - 2(S - X_2) = 2X_2 - S$. Since $S < X_3$, it must be the case that $S < 2X_2$ because otherwise it will become $X_3 > 2X_2$, i.e. $X_3 - X_2 > X_2$, which means $X_2 - X_1 > X_2$. This amounts to saying $X_1 < 0$, which cannot be true. Therefore, the additional cashflow in this case cannot be negative.

If $X_1 < X_2 < X_3 < S$, all calls will be exercised, yielding a cash flow of $(S - X_1) - 2(S - X_2) + (S - X_3) = 2X_2 - X_1 - X_3 = 0$.

Therefore, the cashflows will always be positive regardless the stock price at the maturity date, without taking transaction costs into account.

Because this arbitrage strategy generates instant risk-free profit, we need to examine whether that instant profit is greater than the total transaction costs involved when trying to evaluate its feasibility in real financial market. If yes, an instantaneous risk-free profit can be generated, and such an arbitrage strategy can thus be considered feasible in reality. Assuming that the cost for each buying and selling of the option is the same, say t , the arbitrage strategy, which involves selling two calls and buying two calls will have a total transaction cost of $4t$. Therefore, for the arbitrage strategy to be feasible, we need

$$-C_1 - C_3 + 2C_2 - 4t > 0$$

Note that while entering short positions for call options, the arbitrageur will usually need to also input a certain amount of margin. The margin will not be returned until all positions are closed, and therefore the arbitrageur may theoretically have another additional cost (i.e. the time value of the margin). But in our research for the convexity condition, we ignore this cost because it is usually floating as a result of the uncertain risk-free rate and the exact amount of margin that the arbitrageur needs to put in.

3.3. Put-call Parity Violations

The put-call parity principle requires that portfolios having the same payoff should always share an identical or similar cashflow if the options are European style. When considering European options with dividend-paying stocks, the put-call parity function is modified as $S - PV(DIV) + P = C + PV(X)$. Violations within \pm CNY15 are ignored in this study, with an assumption that CNY15 is the average transaction cost per contract. In other words, situations other than $-CNY15 < S - PV(DIV) + P - C - PV(X) < CNY15$ entail arbitrage opportunities.

3.4. Black-Scholes-Model and Delta Hedge Strategy

Given that the CSI 300 index options are European-style options with dividends, the prices of call and put options are modelled by the below Black-Scholes equations. The notation is carried forward from Table 2.

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_f - \gamma + \frac{\sigma^2}{2}\right) * T}{\sigma / \sqrt{T}}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

$$N(d_1) = \text{NORMSDIST}(d_1) \text{ (Excel function)}$$

$$N(d_2) = \text{NORMSDIST}(d_2) \text{ (Excel function)}$$

The core of a delta-hedge arbitrage strategy is to hedge delta and to ensure that the portfolio is delta neutral and insensitive towards the movement of stock prices. Delta implies the ratio to hedge. More specifically, if the actual call option price is lower than the theoretical price, we will long (short) one call option while selling (buying) delta shares of stock, with a cashflow of $\Delta c * S - C$ ($C - \Delta c * S$). The same approach can be adapted for put options by using the put delta instead.

The transaction costs of selling or buying a call or a put are estimated to be CNY15. The cost during the whole trading period of a mispriced call is estimated to be CNY50, and that of a mispriced put is CNY60. This discrepancy results because the put is less liquid and may generate more costs during the daily hedge period. In addition, we used the historical volatility instead to approximately calculate the fair value of options.

Applying formula (1) and (2) enables us to find the theoretical prices of calls and puts. It is meaningful to find the mispricing between theoretical prices and actual prices because this difference implies the arbitrage opportunities.

$$C = S * e^{-\gamma * T} * N(d_1) - X * e^{-r_f * T} * N(d_2) \quad (1)$$

$$P = X * e^{-r_f * T} * N(-d_2) - S * e^{-\gamma * T} * N(-d_1) \quad (2)$$

Theoretically, a mispricing between actual price and theoretical price is a trading signal for a delta hedge strategy. A mispricing occurs if one of the following formulas is satisfied.

$$C > S * e^{-\gamma * T} * N(d_1) - X * e^{-r_f * T} * N(d_2)$$

$$C < S * e^{-\gamma * T} * N(d_1) - X * e^{-r_f * T} * N(d_2)$$

$$P > X * e^{-r_f * T} * N(-d_2) - S * e^{-\gamma * T} * N(-d_1)$$

$$P < X * e^{-r_f * T} * N(-d_2) - S * e^{-\gamma * T} * N(-d_1)$$

We enter the trade when there is a mispricing. After the first trading day, we implement a daily hedge towards our portfolios as suggested above until the mispricing is eliminated or the option maturity is reached. On the final day, if the actual call option price is lower (higher) than the initial theoretical price, to close the position, we should short (long) one share of call options while buying (selling) delta shares of stock. The cashflow will be $C - \Delta c * S$ ($\Delta c * S - C$). Total profit will be the sum of the daily cashflow (changes in daily money spent on hedging) plus the profit earned on the first day minus the money spent on the closing position and the transaction costs.

Practically, it would be better to enter the trade when the initial mispricing is large enough to cover the transaction costs. Thus, we adjust the trading signal into the formulas below.

$$C > S * e^{-\gamma * T} * N(d_1) - X * e^{-r_f * T} * N(d_2) + 50$$

$$C < S * e^{-\gamma * T} * N(d_1) - X * e^{-r_f * T} * N(d_2) - 50$$

$$P > X * e^{-r_f * T} * N(-d_2) - S * e^{-\gamma * T} * N(-d_1) + 60$$

$$P < X * e^{-r_f * T} * N(-d_2) - S * e^{-\gamma * T} * N(-d_1) - 60$$

4. Findings

The data was divided into three parts based on the moneyness to test whether the moneyness of options influences the violation ratio of boundary, convexity and PCP conditions. The at-the-money option range is set at $0.97 \leq S/X \leq 1.03$ [18].

4.1. Boundary Violation and Arbitrage

As seen in Table 2, the violation ratios vary greatly among different moneyness. CSI 300 index options have a quite huge boundary violation ratio of 7.31% on average, which can be roughly explained by the relatively illiquid market conditions (average daily turnover of CNY 530 million, 500 hands). The put option has a much larger violation ratio 9.17%, which is nearly twice of that of the call option. This result may be explained by a preference towards call options from Chinese investors. Due to the presence of market imperfections, such as short-selling restrictions and disproportionate composition of individual and institution investors, Chinese investors have more opportunities to profit in bullish markets than bearish markets. This natural affinity to bullish markets makes investors set more call positions than put positions, hoping to profit from a market rise. This has led much better liquidity conditions for call options.

The results categorised by moneyness suggest that almost zero boundary condition violations can be found in out-of-the-money option. In-the-money options contribute up to 75% of violations. In particular, the violation rate for in-the-money put options is as high as 22.53%. As for the magnitude of violations, results indicate that despite relatively low violation rate, call options have a larger violation spread, almost 1.5 times higher than put options on average. This means that arbitrageurs will gain more profits from call options than put options.

Table 2. Boundary Violation

Option	Moneyness	N	Violation Ratio	Spread mean	Spread medium	Spread STD
Call	ITM	17131	11.32%	112.11	45.65	130.67
	ATM	8062	0.88%	40.05	34.46	30.83
	OTM	11729	0.01%	11.73	11.73	0
	Total	36923	5.44%	109.56	44.52	129.15
Put	ITM	11729	22.53%	79.71	54.33	85.61
	ATM	8062	9.23%	39.32	31.08	33.45
	OTM	17131	0.01%	54.26	47.73	5.97
	Total	36923	9.17%	70.83	47.88	79.01

4.2. Convexity Violation and Arbitrage

We use Pandas with Python to divide our data into groups, and we are only interested in groups with size greater than or equal to 3 for the sake of comparing rates of change. We examine the options in those groups with consecutive strike prices and see whether their premiums follow the convexity condition. The pandas code can be found in the appendix. The findings obtained from our program are summarized in Table 3.

Table 3. Convexity Violations

Moneyness	Sample Size	Convexity Violation Size	Violation Ratio
ITM	11276	4196	37.212%
OTM	6209	1870	30.118%
Total	20550	6806	33.119%

The sample sizes for in-the-money and out-of-the-money do not sum up to the total sample size. This is because when we are counting all in-the-money cases, we are looking at 3 options whose strike prices are all below the current value of its underlying index; when we are counting all out-of-the-money cases, we are looking at 3 options whose strike prices are all above the current value of its underlying index. However, there are also convexity violations, where 1 or 2 of the 3 options are in-the-money and the rest are out-of-the-money. Their statistics are also easily calculatable:

Total sample size = $20550 - 6209 - 11276 = 3065$

Convexity violation size = $6806 - 1870 - 4196 = 740$

Violation ratio = $740 / 3065 = 24.144\%$

As we can see from Table 4, in general, for the index call options, convexity violations happen roughly in 33% of cases. Moreover, when three calls with consecutive strike prices are all out-of-the-money, they are less likely to violate the convexity condition than the cases where the

calls are in-the-money. From this it seems that when the call for this option is in-the-money, there are a few more arbitrage opportunities to be exploited using convexity condition. The average arbitrage profit, as calculated by our program is in Table 5.

Table 4. Average Convexity Arbitrage Profits (CNY)

Moneyiness	Average Profit
ITM	36.171
OTM	15.861
Total	29.439

Then, we take into consideration the transaction costs as discussed in the previous section. The transaction costs for trading (selling and buying) options for this index future is around 15 CNY per board lot[19]. Therefore, the transaction cost for each butterfly spread strategy is $4 \times 15 = 60$ CNY. With our program, we obtain the following tables:

Table 5. Convexity Violations

Moneyiness	Sample Size	Convexity Violation Size	Violation Ratio
ITM	11276	489	4.337%
OTM	6209	88	1.417%
Total	20550	636	3.095%

Table 6. Average Convexity Arbitrage Profits (CNY)

Moneyiness	Average Profit
ITM	150.034
OTM	122.331
Total	145.869

The two tables above illustrate some interesting results after we take transaction costs into consideration. First, the percentage of violations significantly decreases—no matter it is in the money or out of the money, the violations occur at a rate lower than 5%. Second, however, amongst the few violations we found, the average arbitrage profits are substantially improved in both in the money and out of the money cases. This suggests that there are some evident misprices but to find those opportunities can be hard since the percentage of such violations are rather low. Last but not least, one factor that may contribute to what we have found is the relatively low trading volume of some of the option contracts in our data sets since their prices are more easily affected by the action of individual investors instead of the general market. In other words, our research of the data set suggests that convexity violations do exist, and the average convexity arbitrage profits can be rather considerable.

4.3. Put Call Parity

There is more mispricing in put-call parity strategies when compared to boundary violations and convexity violations. The overall violation ratio is 96.86% and the average violation magnitude is 428.25. Such violations are primarily caused by the call being underprice and the put being overpriced. This is because when the moneyness of call decreases, the average violation magnitude rises dramatically from 104.92 to 495.33. It is reasonable to assume that the put price is likely to be higher and call price is likely to be lower when S/X becomes smaller than 0.97. Yet, such huge deviations are abnormal and may imply an underpricing of calls or an overpricing of puts. Similarly, when the moneyness of call increases, the violation magnitude again experiences an abnormally dramatic increase. This supports the idea that the call might be overpriced and the put might be underpriced.

When categorized by moneyness, the violation ratio is smallest when options are at-the-money and the average violation magnitude is also smallest, 104.92. The violation ratio varies slightly when calls are in-the-money and out-of-the-money. Their spread mean is almost five times higher than the spread mean of at-the-money options, accompanied by a higher standard deviation.

High violation ratios and large spread means imply a significant mispricing and create profitable arbitrage opportunities for traders. The total profit is approximately CNY1,581,171,300 during the sample period. Traders can gain more profits (CNY665,566,100) through trading out-the-money calls or in-the-money puts. This is almost double the profits earned when $S/X < 0.97$, although they share a similar violation frequency.

Table 7.

Moneyness (S/X)	Sample Size	Violation Ratio	Spread Mean	Spread Median	Spread STD	Sum of Profit (CNY)*100
(0, 0.97)	11729	99.16%	495.33	362.26	414.32	5,809,766
[0.97, 1.03]	8062	88.75%	104.92	75.93	113.12	845,895
(1.03, ∞)	17131	99.04%	534.47	421.69	436.09	9,156,052
Total	36922	96.86%	428.25	307.14	418.39	15,811,713

4.4. Delta-Hedge Strategy

Results are divided into four categories based on initial trading day’s mispricing condition.

C.E. = Act.Call is relatively expensive initially

C.C. = Act.Call is relatively cheap initially

P.E. = Act.Put is relatively expensive initially

P.C. = Act.Put is relatively cheap initially

We first implement theoretical delta hedge strategy, where the trading signal is a deviation between actual option price and theoretical option price calculated through BS-Model. Results are summarized in Table 9. Ratio of Trade Enter is the number of days entering the trade divided by N. The frequency of C.C. and P.C. is much higher than C.E. and P.E. However, the ratio of trade entered for C.C. and P.C. is less than C.E. and P.E. This suggests that the hedging period of C.C. and P.C. is longer and may incur more trading fees for daily rebalancing. The average violation magnitude of C.C. and P.C. is more than twice as large as C.E. and P.E., while their standard deviations differ slightly. In a theoretical delta hedge strategy, C.E. is the only setting that generates positive profits (CNY4,623,219) during the sample period.

We then design a model to implement the delta hedge strategy through a more practical method. We adjust the threshold (trading signal). As mentioned in section 3.4, we only enter the trade C.C. and C.E. when the deviation between theoretical price and actual price of call is larger than CNY 50, because it covers the estimated transaction costs. Similarly, we only enter the trade P.C. and P.E. when the deviation between theoretical price and actual price of the put is larger than CNY 60. After adjustment for the trading threshold, the ratio of trade entered decreases. The spread median and spread mean all increase except C.C. condition. Unlike theoretical delta hedge strategy, both C.E. and PE. generate positive profits. Total loss is mitigated but still significant (-CNY8,533,731,200).

Table 8.
Theoretical Delta Hedge Strategy

	N	Ratio of trade entered	Spread Mean	Spread Median	Spread STD	Profit (CNY)*100
C.E.	1636	6.4%	182.51	61.54	254.53	46232.19
C.C.	34697	3.6%	270.27	209.02	253.20	-313005.28
P.E.	4209	4.6%	140.09	76.21	225.56	-885.16
P.C.	32135	3.6%	264.07	189.68	263.74	-800429.72
Total						-1068087.97

Practical Delta Hedge Strategy

	N	Ratio of trade entered	Spread Mean	Spread Median	Spread STD	Profit (CNY)*100
C.E.	795	4.9%	216.49	79.20	283.50	16987.56
C.C.	33841	3.5%	267.80	207.04	253.97	-61736.29
P.E.	3624	4.0%	227.21	146.66	242.15	2015.92
P.C.	31166	3.5%	270.15	197.33	264.88	-810640.33
Total						-85,337,312

It is notable that there is a significant loss towards delta hedge strategy. This is consistent with multiple previous study conducted by previous researchers[20]. From the table both put and call are more likely to be underpriced than overpriced when we enter the trade. However, during the trading period, the prices of put and call are more likely to be higher than theoretical price. There is a huge deviation between the actual option price and theoretical option price, particularly for put that is cheaper than theoretical value initially.

5. Conclusion

The paper examined boundary violations, convexity violations, put-call parity violations in CSI 300 Index market from 2015 to 2020. Black-Scholes Model and delta hedge strategy are also adopted to exploit the mispricing. The paper also categorized these violations by moneyness and analyzed their profitability. Overall, Chinese option market is still less mature accompanied with many mispricing and violations.

There is no upper violation during the sample period, which is common to many markets. In terms of lower bound violations, the frequency of mispricing is higher for put options. 9.17% of 36,923 put option observations violate the lower bound rule while 5.44% of 36,923 call

option observations violate the rule. Violation ratio decreases as moneyness decreases. The median of mispricing magnitude is also positively associated with moneyness. The frequency of mispricing is higher in convexity violations. 33.12% of 20,550 observations violate convexity conditions and it more likely occurs on in-the-money option. Average convexity arbitrage profit excluding transaction costs is CNY 29.439. When taking transaction cost into consideration, the percentage of convexity violations significantly decreases to 3.095%, whereas the average arbitrage profit increases to CNY 145.87.

There are considerable mispricing conditions in put-call parity strategy. 96.8% of 36,922 put-call pairs violate the put-call parity principle. Total profit is CNY 1,581,171,300 during the sample period. In contrast, delta hedge strategy suffered a significant loss during the sample period, -¥1068087.97 for theoretical strategy. However, after adjusting the threshold (trading signal), the loss reduced to -¥85,337,300, although it is still not ideal. Therefore, it is not recommended to implement delta hedge strategy in CSI index option market.

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