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A New Trapezoidal Fuzzy Linguistic Induced Generalized OWA Operator :A Case Study for the Food Supply

Yue Xiao [a], Kaishun Chen

(Affiliation): Iflytek artificial intelligence institute,Rongzhi College of Chongqing Technology And Business University, Chongqing, China

[a]1126893227@qq.com

Abstract

With the development of China's social and economic development, food safety has gradually attracted more and more attention. The variety of food is more and more abundant. Hence, it's particularly important to choose the right supplier. To deal with this problem, we present the trapezoidal fuzzy linguistic induce generalized ordered weighted averaging (TrFLIGOWA) operator to aggregate the food supply and we end the paper with a numerical example about food supply is given to illustrate the practicality and effectiveness of the proposed operator in the multiple attribute group decision making.

Keywords

The food supply; Induce ordered weighted averaging operator; Trapezoidal fuzzy linguistic induce generalized ordered weighted averaging operator.

1. Introduction

With the development of China's social and economic development, food safety has gradually attracted more and more attention. The variety of food is more and more abundant. New food safety problems keep emerging, which seriously harm the health of the people. Therefore, it's particularly important to choose the right supplier. Hence, Efficient quantitative expression for linguistic information is critical to deal with the multiple attribute group decision making (MAGDM), In the process of decision making, a decision maker needs to compare a set of alternatives to give his/her preference. Xian[1] presented the trapezoidal Pythagorean fuzzy linguistic variable to show the location information. In order to comprehensively select the most suitable supplier, the most important thing is that how to aggregate these decision-making information. Many scholars have proposed different aggregation operators to process the fuzzy linguistic information and to sort the alternatives. Yager[2,3] presented the ordered weighted averaging (OWA) operator. Subsequently, the researchers extended the OWA operator and proposed the induce generalized OWA(IGOWA) operator[4]. However, in some case, OWA, GOWA, IGOWA and other operators can't effectively solve complex Pythagorean fuzzy linguistic information. Therefore, how to expand GOWA and IGOWA operator to deal with complex Pythagorean fuzzy linguistic information is a significant work, because they have a strong advantage in multiple attribute group decision making.

2. Preliminaries

In this chapter, we will briefly review the basic concepts of the fuzzy linguistic values, the IOWA operator, the IGOWA operator for the convenience of analysis and these are used throughout the paper.

2.1. The Expression of Linguistic Information

Definition 1. Suppose that a set of linguistic labels $\tilde{S} = \{s_{\tilde{1}}, s_{\tilde{2}}, ..., s_{\tilde{T}}\}$ that be the pre-established finite and totally ordered discrete linguistic term set, where $s_{\tilde{i}}$ denotes the $\tilde{i}$ th linguistic term of $\tilde{S}$ and $\tilde{T}$ represents the cardinality of $\tilde{S}$. For example, a set of nine terms S could be represented as follows[5,6]:

$$S = \{s_{\tilde{9}} = perfect(P), s_{\tilde{8}} = extremly\,high(EH), s_{\tilde{7}} = very\,high(V),$$
$$s_{\tilde{6}} = high(H), s_{\tilde{5}} = medium(M), s_{\tilde{4}} = low(L), s_{\tilde{3}} = very\,low(VL),$$
$$s_{\tilde{2}} = extrymly\,low(EL), s_{\tilde{1}} = none(N)\}$$

in which $s_{\tilde{i}} < s_{\tilde{j}}$ iff $\tilde{i} < \tilde{j}$. Usually, in these cases, it is often required that the linguistic term set satisfies the following additional characteristics[7,8,9,10]:

(1) There is a negation operator, e.g., $Neg(s_{\tilde{i}}) = s_{\tilde{j}}, \tilde{i} = \tilde{T} - 1(\tilde{T}+1$ is the cardinality).

(2) Maximization and Minimization operator: $Max(s_{\tilde{i}}, s_{\tilde{j}}) = s_{\tilde{i}}$, if $s_{\tilde{i}} \geq s_{\tilde{j}}$, $Min(s_{\tilde{i}}, s_{\tilde{j}}) = s_{\tilde{i}}$ if $s_{\tilde{i}} \leq s_{\tilde{j}}$.

2.2. Trapezoidal Fuzzy Number

Definition 2. A trapezoidal fuzzy number $\tilde{a}$ can be defined by a triplet $\tilde{a} = (a_1, a_2, a_3, a_4)$, $a_1, a_2, a_3, a_4 \in R$. The membership function[11] $\mu_{\tilde{a}}(x)$ is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & Otherwise. \end{cases} \quad (1)$$

If $a_2 = a_3$ in trapezoidal intuitionistic fuzzy number, we have the triangular intuitionistic fuzzy number as special case of the trapezoidal intuitionistic fuzzy numbers.

Definition 3. Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be any two positive trapezoidal fuzzy number, $\lambda \in R, \kappa \in R$, the basic operational laws related to trapezoidal fuzzy numbers are as follows [12]:

$$\begin{aligned} &(1)\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \\ &(2)\tilde{a} \otimes \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4) \\ &(3)\lambda \odot \tilde{a} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4) \\ &(4)\tilde{a}^{\kappa} = (a_1^{\kappa}, a_2^{\kappa}, a_3^{\kappa}, a_4^{\kappa}) \\ &(5)\frac{1}{\tilde{a}} = (\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}). \end{aligned} \quad (2)$$

Note that the results of Eqs. (4)-(5) are not TrFNs, but these results can be approximated by TrFNs.

Definition 4. Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ the positive trapezoidal fuzzy number, the graded mean value representation of a trapezoidal fuzzy number $\tilde{a}$ is defined as[3]:

$$P(\tilde{a})=\frac{a_1+2a_2+2a_3+a_4}{6} \tag{3}$$

Definition 5. Let $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$ be any two positive trapezoidal fuzzy number, if $P(\tilde{a})\le P(\tilde{b})$, then $\tilde{a}\le\tilde{b}$.

The fuzzy linguistic scale S is used to evaluate the performance of decisions, and fuzzy linguistic terms are denoted by trapezoidal fuzzy numbers as follows:

$$S=\{s_{\tilde{9}}=s_{(0.7,0.8,0.9,1)}, s_{\tilde{8}}=s_{(0.6,0.7,0.8,0.9)}, s_{\tilde{7}}=s_{(0.5,0.6,0.7,0.8)},$$
$$s_{\tilde{6}}=s_{(0.4,0.5,0.6,0.7)}, s_{\tilde{5}}=s_{(0.3,0.4,0.5,0.6)}, s_{\tilde{4}}=s_{(0.2,0.3,0.4,0.5)},$$
$$s_{\tilde{3}}=s_{(0.1,0.2,0.3,0.4)}, s_{\tilde{2}}=s_{(0,0.1,0.2,0.3)}\}, s_{\tilde{1}}=s_{(0,0.1,0.1,0.2)}$$

Definition 6. Let $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$ be any two positive trapezoidal fuzzy number, $s_{\tilde{a}},s_{\tilde{b}}\in\tilde{S}$, if $\tilde{a}\le\tilde{b}$, then $s_{\tilde{a}}\le s_{\tilde{b}}$.

According to Definition 6, there is obviously $s_{\tilde{9}}>s_{\tilde{8}}>s_{\tilde{7}}>s_{\tilde{6}}>s_{\tilde{5}}>s_{\tilde{4}}>s_{\tilde{3}}>s_{\tilde{2}}>s_{\tilde{1}}$. The operational laws as follows[13]:

$$\begin{aligned}
&(1) s_{\tilde{\alpha}}\oplus s_{\tilde{\beta}}=s_{\tilde{\alpha}+\tilde{\beta}}=s_{\tilde{\beta}}\oplus s_{\tilde{\alpha}}\\
&(2) y\odot s_{\tilde{\alpha}}=s_{y\tilde{\alpha}}\\
&(3) y\odot(s_{\tilde{\alpha}}\oplus s_{\tilde{\beta}})=y\odot s_{\tilde{\alpha}}\oplus y\odot s_{\tilde{\beta}}=s_{y\tilde{\alpha}+y\tilde{\beta}}\\
&(4) (y+z)\odot s_{\tilde{a}}=y\odot s_{\tilde{a}}\oplus z\odot s_{\tilde{a}}=s_{(y+z)\tilde{a}}\\
&(5) s_{\tilde{a}}\otimes s_{\tilde{b}}=s_{(a_1\times b_1,a_2\times b_2,a_3\times b_3,a_4\times b_4)}\\
&(6) s_{\tilde{a}}^{\kappa}=s_{\tilde{a}^{\kappa}}=s_{(a_1^{\kappa},a_2^{\kappa},a_3^{\kappa},a_4^{\kappa})}\\
&(7) \frac{1}{s_{\tilde{a}}}=s_{(\frac{1}{a_4},\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1})}.
\end{aligned} \tag{4}$$

2.3. The IOWA Operator and the IGOWA Operator

On the basis of Mitchell's work[14], Yager [3] proposed the Induced Ordered Weighted Averaging (IOWA) operator as an extension of OWA operator:

Definition 7. The n-dimension IOWA operator is a function can be expressed as $\Phi_{IOWA}:R^n\times R^n\to R$, to which a weighting vector is associated $W=(w_1,w_2,\ldots,w_n)$, such that $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$, and it is defined to aggregate the set of second arguments of a list of n pairs $\{(u_1,p_1),(u_2,p_2),\ldots,(u_n,p_n)\}$ according to the following expression:

$$\Phi_{IOWA}((u_1,p_1),(u_2,p_2),\ldots,(u_n,p_n))=\sum_{i=1}^{n}w_ip_{\sigma(i)} \tag{5}$$

where $\sigma:(1,2,\ldots,n)\to(1,2,\ldots,n)$ is a permutation such that $u_{\sigma(i)}\ge u_{\sigma(i+1)}\ \forall i=1,2,\ldots,n-1$, that is, $(u_{\sigma(i)},p_{\sigma(i)})$ is the pair with $u_{\sigma(i)}$ the ith highest value in the set $\{u_1,u_2,\ldots,u_n\}$. $p_1,p_2,\ldots,p_n$ is induced by the ordering of the values $u_1,u_2,\ldots,u_n$ associated with them. The set of values $u_1,u_2,\ldots,u_n$ called order inducing variable and $p_1,p_2,\ldots,p_n$ the values of the argument variable[15].

Xian[4] proposed the definition of the IGOWA operator is as follows:

Definition 8. An IGOWA operator of dimension n is a mapping $\Phi_{IGOWA}: R^n \times R^n \to R$ that has an associated weighting vector w of dimension n such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, then:

$$\Phi_{IGOWA}((u_1, p_1), (u_2, p_2), \ldots, (u_n, p_n)) = (\sum_{i=1}^{n} w_i p_{\sigma(i)}^{\lambda})^{\frac{1}{\lambda}} \tag{6}$$

Where $p_{\sigma(i)}$ is the $p_1, p_2, \ldots, p_n$ value of the IGOWA pair u_i, $p_{\sigma(i)}^{\lambda}$ having the i th largest u_i, $u_1, u_2, \ldots, u_n$ is the order inducing variable, $p_{\sigma(i)}^{\lambda}$ is the argument variable , λ is a parameter and $\lambda \in (-\infty, \infty)$.

We can clearly see that if $\lambda = 1$, we can get the IOWA operator. If $\lambda = 0$, the IGOWA operator is reduced to the IOWG operator, if $\lambda = -1$, the IGOWA operator is reduced to the IOWHA operator and if $\lambda = 2$, the IGOWA operator is reduced to the IOWQA operator.

3. The Trapezoidal Fuzzy Linguistic Induced Generalized Ordered Weighted Averaging (TrFLIGOWA) Operator

However, IGOWA operator can only aggregate real numbers, can't effectively deal with complex fuzzy linguistic information. We develop the trapezoidal fuzzy linguistic induced generalized ordered weighted averaging (TrFLIGOWA) operator which is an extension of the IGOWA operator that can solve complex fuzzy linguistic information in the aggregation represented in the form of fuzzy linguistic labels. In our real life, there is uncertainty in the decision-making factors affecting us, and the traditional variables cannot express these decision-making information clearly, thus, we used fuzzy linguistic variables to express the uncertainty of these information. The TrFLIGOWA operator mainly uses the characteristics of the IGOWA and fuzzy linguistic information to aggregate the decision-making information and sort the results by using order inducing variables. For a collection of fuzzy linguistic labels, it can be defined as follows.

Definition 9. An n-dimensional trapezoidal fuzzy linguistic induced generalized ordered weighted averaging (TrFLIGOWA) operator is a function can be expressed as $\Phi_{TrFLIGOWA}: R^n \times \tilde{S}^n \to \tilde{S}$, and the weighting vector $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. It is defined to aggregate a list of values $(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$ according to the following expression:

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (\bigoplus_{i=1}^{n} w_i \odot s_{\tilde{\beta}_i}^{\lambda})^{\frac{1}{\lambda}} = s_{\tilde{\beta}} \tag{7}$$

where $\tilde{\beta} = (\sum_{i=1}^{n} w_i \tilde{\beta}_i^{\lambda})^{\frac{1}{\lambda}}$, $s_{\tilde{\beta}_i}$ is $s_{\tilde{\alpha}_i}$ value of the TrFLIGOWA pair $(u_i, s_{\tilde{\alpha}_i})$ having the i th largest u_i, u_i is the order inducing variable and $s_{\tilde{\alpha}_i}$ is the argument variable represented in the form of individual linguistic value, so $(s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}, \ldots, s_{\tilde{\beta}_n}) \to (s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$ being a permutation and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

In terms of the generalized reorder step, we can distinguish between the descending TrFLIGOWA (DTrFLIGOWA) operator and the ascending TrFLIGOWA (ATrFLIGOWA) operator by using $w_i = w_{n-i+1}^{*}$, where w_i is the i th weight of the DTrFLIGOWA and w_{n-i+1}^{*} the i th weight of the ATrFLIGOWA operator. Note that if the weighting vector is not normalized, $W = \sum_{i=1}^{n} w_i \neq 1$, then, the TrFLIGOWA operator can be expressed as:

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (\frac{1}{W} \bigoplus_{i=1}^{n} w_i \odot s_{\tilde{\beta}_i}^{\lambda})^{\frac{1}{\lambda}} = s_{\tilde{\beta}} \quad (8)$$

The TrFLIGOWA operator has the following properties.

Theorem 1 (Commutativity) Let $((u_1^*, s_{\tilde{\alpha}_1}^*), (u_2^*, s_{\tilde{\alpha}_2}^*), \ldots, (u_n^*, s_{\tilde{\alpha}_n}^*))$ is any permutation of the fuzzy linguistic scale data vector $((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$, then:

$$\Phi_{TrFLIGOWA}((u_1^*, s_{\tilde{\alpha}_1}^*), (u_2^* s_{\tilde{\alpha}_2}^*), \ldots, (u_n^* s_{\tilde{\alpha}_n}^*))$$
$$= \Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$$

Proof: Let

$$\Phi_{TrFLIGOWA}((u_1^*, s_{\tilde{\alpha}_1}^*), (u_2^*, s_{\tilde{\alpha}_2}^*), \ldots, (u_n^*, s_{\tilde{\alpha}_n}^*))$$
$$= (\bigoplus_{i=1}^{n} w_i \odot s_{\tilde{\beta}_i}^{*\lambda})^{\frac{1}{\lambda}} = s_{\widetilde{\beta^*}},$$

$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (\bigoplus_{i=1}^{n} w_i \odot s_{\tilde{\beta}_i}^{\lambda})^{\frac{1}{\lambda}} = s_{\tilde{\beta}}$ Since $\widetilde{\beta^*} = (\sum_{i=1}^{n} w_i \tilde{\beta}_i^{*\lambda})^{\frac{1}{\lambda}}$, $\tilde{\beta} = (\sum_{i=1}^{n} w_i \tilde{\beta}_i^{\lambda})^{\frac{1}{\lambda}}$ and $((u_1^*, s_{\tilde{\alpha}_1}^*), (u_2^*, s_{\tilde{\alpha}_2}^*), \ldots, (u_n^*, s_{\tilde{\alpha}_n}^*))$ is a permutation of data vector $((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$, we have $\beta_i = \beta_i^*$, $i = 1, 2, \ldots, n$, then $s_{\tilde{\beta}_i} = s_{\tilde{\beta}_i}^*$, so

$$\Phi_{TrFLIGOWA}((u_1^*, s_{\tilde{\alpha}_1}^*), \ldots, (u_n^* s_{\tilde{\alpha}_n}^*))$$
$$= \Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$$

Theorem 2 (Idempotency**)** If $s_{\widetilde{\alpha_i}}, s_{\tilde{\alpha}} \in \tilde{S}$ and $\widetilde{\alpha_i} = \tilde{\alpha}, i \in N$, then

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = s_{\tilde{\alpha}}$$

Proof: Since $\widetilde{\alpha_i} = \tilde{\alpha}$ for all i, according the Definition 6, $s_{\widetilde{\alpha_i}} = s_{\tilde{\alpha}}$ for all i then

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$$
$$= (w_1 \odot s_{\tilde{\beta}_1}^{\lambda} \oplus w_2 \odot s_{\tilde{\beta}_2}^{\lambda} \oplus \ldots \oplus w_n \odot s_{\tilde{\beta}_n}^{\lambda})^{\frac{1}{\lambda}}$$

$$= w_1 \odot s_{\tilde{\alpha}}^{\lambda} \oplus w_2 \odot s_{\tilde{\alpha}}^{\lambda} \oplus \ldots \oplus w_n \odot s_{\tilde{\alpha}}^{\lambda})^{\frac{1}{\lambda}}$$

$$= ((w_1 + w_2 + \ldots + w_n) \odot s_{\tilde{\alpha}}^{\lambda})^{\frac{1}{\lambda}} = s_{\tilde{\alpha}}.$$

Theorem 3 (Monotonicity**)** Let $((u_1, s_{\tilde{\alpha}_1}^*), (u_2, s_{\tilde{\alpha}_2}^*), \ldots, (u_n, s_{\tilde{\alpha}_n}^*))$ and $((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n}))$, are two data vector, if $\widetilde{\alpha_i} \le \widetilde{\alpha_i}^*$, $s_{\widetilde{\alpha_i}}, s_{\widetilde{\alpha_i}^*} \in \tilde{S}$, for all i, then

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n})) \le \Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}^*), \ldots, (u_n, s_{\tilde{\alpha}_n}^*))$$

Proof: Since $\widetilde{\alpha_i} \le \widetilde{\alpha_i}^*$ for all i, according the Definition 6, then $s_{\widetilde{\alpha_i}} \le s_{\widetilde{\alpha_i}^*}$, for all $i \in N$. Let:

$$\Phi_{TrFLIGOWA}((u_1, s^*_{\tilde{\alpha}_1}), (u_2, s^*_{\tilde{\alpha}_2}), \ldots, (u_n, s^*_{\tilde{\alpha}_n}))$$
$$= (\bigoplus_{i=1}^n w_i \odot s^{*\lambda}_{\tilde{\beta}_i})^{\frac{1}{\lambda}} = s_{\widetilde{\beta^*}}$$

$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (\bigoplus_{i=1}^n w_i \odot s^{\lambda}_{\tilde{\beta}_i})^{\frac{1}{\lambda}} = s_{\tilde{\beta}}$,where $\widetilde{\beta^*} = (\sum_{i=1}^n w_i \tilde{\beta}_i^{*\lambda})^{\frac{1}{\lambda}}$, $\tilde{\beta} = (\sum_{i=1}^n w_i \tilde{\beta}_i^{\lambda})^{\frac{1}{\lambda}}$, since $(s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}, \ldots, s_{\tilde{\beta}_n})$ is a permutation of $(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, and $(s_{\tilde{\beta}^*_1}, s_{\tilde{\beta}^*_2}, \ldots, s_{\tilde{\beta}^*_n})$ is a permutation of $(s_{\tilde{\alpha}^*_1}, s_{\tilde{\alpha}^*_2}, \ldots, s_{\tilde{\alpha}^*_n})$ respectively , so, we have $s_{\tilde{\beta}_i} \le s_{\tilde{\beta}^*_i}$ for all i, then $s_{\tilde{\beta}} \le s_{\tilde{\beta}^*}$.

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n})) \le \Phi_{TrFLIGOWA}((u_1, s^*_{\tilde{\alpha}_1}), \ldots, (u_n, s^*_{\tilde{\alpha}_n}))$$

Theorem 4 (Boundedness) Let $s_{\tilde{\alpha}} = \min_i(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, $s_{\tilde{\beta}} = \max_i(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, then

$$s_{\tilde{\alpha}} \le \Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) \le s_{\tilde{\beta}}$$

Proof: Since $s_{\tilde{\alpha}} \le s_{\widetilde{\alpha_i}} \le s_{\tilde{\beta}}$ for all i, using the theorem 2-3, then:

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (w_1 \odot s^{\lambda}_{\tilde{\beta}_1} \oplus w_2 \odot s^{\lambda}_{\tilde{\beta}_2} \oplus \ldots \oplus w_n \odot s^{\lambda}_{\tilde{\beta}_n})^{\frac{1}{\lambda}}$$
$$\ge (w_1 \odot s^{\lambda}_{\tilde{\alpha}} \oplus w_2 \odot s^{\lambda}_{\tilde{\alpha}} \oplus \ldots \oplus w_n \odot s^{\lambda}_{\tilde{\alpha}})^{\frac{1}{\lambda}}$$
$$= ((w_1 + w_2 + \ldots + w_n) \odot s^{\lambda}_{\tilde{\alpha}})^{\frac{1}{\lambda}} = s_{\tilde{\alpha}}.$$

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (w_1 \odot s^{\lambda}_{\tilde{\beta}_1} \oplus w_2 \odot s^{\lambda}_{\tilde{\beta}_2} \oplus \ldots \oplus w_n \odot s^{\lambda}_{\tilde{\beta}_n})^{\frac{1}{\lambda}}$$
$$\le (w_1 \odot s^{\lambda}_{\tilde{\beta}} \oplus w_2 \odot s^{\lambda}_{\tilde{\beta}} \oplus \ldots \oplus w_n \odot s^{\lambda}_{\tilde{\beta}})^{\frac{1}{\lambda}}$$
$$= ((w_1 + w_2 + \ldots + w_n) \odot s^{\lambda}_{\tilde{\beta}})^{\frac{1}{\lambda}} = s_{\tilde{\beta}}$$

So $s_{\tilde{\alpha}} \le \Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), \ldots, (u_n, s_{\tilde{\alpha}_n}) \le s_{\tilde{\beta}}$

The TrFLIGOWA operator provides a parameterized family of aggregation operators. Hence, we distinguish between the families found in the weighting vector W and those found in the parameter λ.

Remark 1. If $\lambda = 1$, we can get the TrFLIOWA operator.

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = \bigoplus_{i=1}^n w_i \odot s_{\tilde{\beta}_i} = s_{\tilde{\beta}} \tag{9}$$

Note that if $w = (w_1, w_2, \ldots, w_n)^T = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, we get the Trapezoidal fuzzy linguistic extended arithmetical averaging (TrFLEAA) operator, if $\alpha_{i1} = \alpha_{i2} = \alpha_{i3}$, that is ,$\widetilde{\alpha_i} = \alpha_i$, the TrPFLIGOWA is reduced to the extended order weighted averaging (EOWA) operator. if $u_i > u_{i+1}$ for all i, we can get the trapezoidal fuzzy linguistic weighted arithmetical averaging

(TrFLWAA) operator. If the ordered position of u_i is the same as the ordered position of $s_{\tilde{\alpha}_i}$, we can get the trapezoidal fuzzy linguistic ordered weighted averaging(TrFLOWA) operator .

Remark 2. If $\lambda = 2$, apparently, we can get the TrFLIEOWA operator.

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = (\bigoplus_{i=1}^{n} w_i \odot s_{\tilde{\beta}_i}^2)^{\frac{1}{2}} = s_{\tilde{\beta}} \tag{10}$$

Note that if $w = (w_1, w_2, \ldots, w_n)^T = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, we get the Trapezoidal fuzzy linguistic normalized Euclidean averaging (TrFLNEA) operator. If $u_i > u_{i+1}$ for all i, we get the trapezoidal fuzzy linguistic Euclidean weighted averaging (TrFLEWA) operator. If the ordered position of u_i is the same as the ordered position of $s_{\tilde{\alpha}_i}$, we get the trapezoidal fuzzy linguistic Euclidean ordered weighted averaging (TrFLEOWA) operator.

Remark 3 . If $\lambda = 0$, apparently, we get the TrFLIOWGA operator.

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = \bigodot_{i=1}^{n} s_{\tilde{\beta}_i}^{w_i} \tag{11}$$

Remark 4. If $\lambda = -1$, then, we get the TrFLIOWHA operator.

$$\Phi_{TrFLIGOWA}((u_1, s_{\tilde{\alpha}_1}), (u_2, s_{\tilde{\alpha}_2}), \ldots, (u_n, s_{\tilde{\alpha}_n})) = \frac{1}{\bigoplus_{i=1}^{n} \frac{w_i}{s_{\tilde{\beta}_i}}} \tag{12}$$

It be note worthy that if $w_i = \frac{1}{n}$ for all $s_{\tilde{a}_i}$, we get the trapezoidal fuzzy linguistic normalized harmonic averaging (TrFLNHA) operator. If $u_i > u_{i+1}$ for all i, we get the trapezoidal fuzzy linguistic weighted harmonic averaging (TrFLWHA) operator. If the ordered position of u_i is the same as the ordered position of $s_{\tilde{\alpha}_i}$, we get the trapezoidal fuzzy linguistic ordered weighted harmonic averaging (TrFLOWHA) operator.

4. Numerical Example

The TrFLIGOWA operator can be widely used in data statistics, control engineering, management science and other fields . In a word, this TrFLIGOWA operator can take full the fuzzy linguistic information of multiple attribute group decision-making into account. In this part, we will deal with a practical numerical example of multiple attribute decision-making problem by using the TrFLIGOWA operator . Supposed a supermarket wants to choose the best supplier that satisfy the needs of the supermarket. They have six supplier to be chosen: x_1 : supplier a ; x_2 : supplier b ; x_3 : supplier c ; x_4 : supplier d ; x_5 : supplier e ; x_6 : supplier f . In order to evaluate these supplier, the group of experts give an overall assessment decision information in Table 1.

Step 1. Constructing a trapezoidal fuzzy linguistic scale decision making matrix $\tilde{S} = ((u_i, s_{\tilde{\alpha}_i})_j)_{k \times k}$. Supposed that the experts compare these six alternatives according to their own evaluation criteria to obtain the trapezoidal Pythagorean fuzzy linguistic induced variable and the fuzzy linguistic decision matrix $\tilde{S}=((u_i,s_{\tilde{\alpha}_i})_j)_{6\times 6}$ is in the table 1[16].

Table 1. Trapezoidal fuzzy linguistic induced decision matrix.

	$(u_1,s_{\tilde{\alpha}_1})$	$(u_2,s_{\tilde{\alpha}_2})$	$(u_3,s_{\tilde{\alpha}_3})$	$(u_4,s_{\tilde{\alpha}_4})$	$(u_5,s_{\tilde{\alpha}_5})$	$(u_6,s_{\tilde{\alpha}_6})$
x_1	$(15,s_{\tilde{6}})$	$(12,s_{\tilde{6}})$	$(17,s_{\tilde{8}})$	$(13,s_{\tilde{6}})$	$(10,s_{\tilde{8}})$	$(11,s_{\tilde{5}})$
x_2	$(17,s_{\tilde{5}})$	$(20,s_{\tilde{7}})$	$(15,s_{\tilde{6}})$	$(13,s_{\tilde{6}})$	$(16,s_{\tilde{6}})$	$(12,s_{\tilde{7}})$
x_3	$(11,s_{\tilde{4}})$	$(14,s_{\tilde{4}})$	$(12,s_{\tilde{9}})$	$(18,s_{\tilde{5}})$	$(13,s_{\tilde{7}})$	$(15,s_{\tilde{8}})$
x_4	$(10,s_{\tilde{7}})$	$(19,s_{\tilde{6}})$	$(17,s_{\tilde{7}})$	$(18,s_{\tilde{5}})$	$(13,s_{\tilde{4}})$	$(14,s_{\tilde{6}})$
x_5	$(12,s_{\tilde{5}})$	$(14,s_{\tilde{6}})$	$(16,s_{\tilde{7}})$	$(17,s_{\tilde{6}})$	$(11,s_{\tilde{5}})$	$(13,s_{\tilde{7}})$
x_6	$(9,s_{\tilde{4}})$	$(11,s_{\tilde{6}})$	$(13,s_{\tilde{8}})$	$(12,s_{\tilde{3}})$	$(7,s_{\tilde{6}})$	$(10,s_{\tilde{6}})$

Step 2. Calculating the overall trapezoidal fuzzy induced values of the alternative by utilizing the TrFLIGOWA operator when $\lambda=2$.

$$s^1_{\tilde{\alpha}_1}=\Phi_{TrFLIGOWA}((15,s_{\tilde{6}}),(12,s_{\tilde{6}}),(17,s_{\tilde{8}}),(13,s_{\tilde{6}}),(10,s_{\tilde{8}}),(11,s_{\tilde{5}}))$$
$$=s_{(0.2130,0.3130,0.4330,0.5730)}$$
$$s^1_{\tilde{\alpha}_2}=\Phi_{TrFLIGOWA}((17,s_{\tilde{5}}),(20,s_{\tilde{7}}),(15,s_{\tilde{6}}),(14,s_{\tilde{7}}),(16,s_{\tilde{6}}),(12,s_{\tilde{7}}))$$
$$=s_{(0.1820,0.2760,0.3900,0.5240)}$$
$$s^1_{\tilde{\alpha}_3}=\Phi_{TrFLIGOWA}((11,s_{\tilde{4}}),(14,s_{\tilde{4}}),(12,s_{\tilde{9}}),(18,s_{\tilde{5}}),(13,s_{\tilde{7}}),(15,s_{\tilde{8}}))$$
$$=s_{(0.2010,0.2930,0.4050,0.4650)}$$
$$s^1_{\tilde{\alpha}_4}=\Phi_{TrFLIGOWA}((10,s_{\tilde{7}}),(19,s_{\tilde{6}}),(17,s_{\tilde{7}}),(15,s_{\tilde{7}}),(13,s_{\tilde{4}}),(14,s_{\tilde{6}}))$$
$$=s_{(0.1930,0.2890,0.4050,0.5410)}$$
$$s^1_{\tilde{\alpha}_5}=\Phi_{TrFLIGOWA}((12,s_{\tilde{5}}),(14,s_{\tilde{6}}),(16,s_{\tilde{7}}),(17,s_{\tilde{6}}),(11,s_{\tilde{5}}),(13,s_{\tilde{7}}))$$
$$=s_{(0.1820,0.2760,0.3900,0.5240)}$$
$$s^1_{\tilde{\alpha}_6}=\Phi_{TrFLIGOWA}((9,s_{\tilde{4}}),(11,s_{\tilde{6}}),(13,s_{\tilde{8}}),(12,s_{\tilde{3}}),(7,s_{\tilde{6}}),(10,s_{\tilde{6}}))$$
$$=s_{(0.1580,0.2310,0.3420,0.4640)}$$

Calculating the overall trapezoidal fuzzy values of the alternative by using the TrFLIOWA operator when $\lambda=1$.

$$s^2_{\tilde{\alpha}_1}=\Phi_{TrFLIOWA}((15,s_{\tilde{6}}),(12,s_{\tilde{6}}),(17,s_{\tilde{8}}),(13,s_{\tilde{6}}),(10,s_{\tilde{8}}),(11,s_{\tilde{5}}))$$
$$=s_{(0.4500,0.5500,0.6500,0.7500)}$$
$$s^2_{\tilde{\alpha}_2}=\Phi_{TrFLIOWA}((17,s_{\tilde{5}}),(20,s_{\tilde{7}}),(15,s_{\tilde{6}}),(14,s_{\tilde{7}}),(16,s_{\tilde{6}}),(12,s_{\tilde{7}}))$$
$$=s_{(0.4200,0.5200,0.6200,0.7200)}$$
$$s^2_{\tilde{\alpha}_3}=\Phi_{TrFLIOWA}((11,s_{\tilde{4}}),(14,s_{\tilde{4}}),(12,s_{\tilde{9}}),(18,s_{\tilde{5}}),(13,s_{\tilde{7}}),(15,s_{\tilde{8}}))$$
$$=s_{(0.4100,0.5100,0.6100,0.7100)}$$

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$$s_{\tilde{\alpha}_4}^2 = \Phi_{TrFLIOWA}((10, s_{\tilde{7}}), (19, s_{\tilde{6}}), (17, s_{\tilde{7}}), (15, s_{\tilde{7}}), (13, s_{\tilde{4}}), (14, s_{\tilde{6}}))$$
$$= s_{(0.4300,0.5300,0.6300,0.7300)}$$
$$s_{\tilde{\alpha}_5}^2 = \Phi_{TrFLIOWA}((12, s_{\tilde{5}}), (14, s_{\tilde{6}}), (16, s_{\tilde{7}}), (17, s_{\tilde{6}}), (11, s_{\tilde{5}}), (13, s_{\tilde{7}}))$$
$$= s_{(0.4200,0.5200,0.6200,0.7200)}$$
$$s_{\tilde{\alpha}_6}^2 = \Phi_{TrFLIOWA}((9, s_{\tilde{4}}), (11, s_{\tilde{6}}), (13, s_{\tilde{8}}), (12, s_{\tilde{3}}), (7, s_{\tilde{6}}), (10, s_{\tilde{6}}))$$
$$= s_{(0.3600,0.4600,0.5600,0.6600)}$$

Step 3. The comparison results obtained by **c**alculating Trapezoidal fuzzy linguistic induced variable as follows:

Table 2. Results comparison analysis

	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
$P(\tilde{\alpha}_1)$	0.6000	0.3797	0.2475
$P(\tilde{\alpha}_2)$	0.5700	0.3397	0.2103
$P(\tilde{\alpha}_3)$	0.5600	0.3437	0.2472
$P(\tilde{\alpha}_4)$	0.5800	0.3537	0.2241
$P(\tilde{\alpha}_5)$	0.5700	0.3397	0.2103
$P(\tilde{\alpha}_6)$	0.5100	0.2947	0.1890

then we will obtain Table 3:

Table 3. Ranking

$\lambda = 1$	$\tilde{\alpha}_1 > \tilde{\alpha}_4 > \tilde{\alpha}_2 = \tilde{\alpha}_5 > \tilde{\alpha}_3 > \tilde{\alpha}_6$
$\lambda = 2$	$\tilde{\alpha}_1 > \tilde{\alpha}_4 > \tilde{\alpha}_3 > \tilde{\alpha}_5 = \tilde{\alpha}_2 > \tilde{\alpha}_6$
$\lambda = 3$	$\tilde{\alpha}_1 > \tilde{\alpha}_3 > \tilde{\alpha}_4 > \tilde{\alpha}_5 = \tilde{\alpha}_2 > \tilde{\alpha}_6$

Step 4. Ranking all the alternatives $x_i (i = 1, 2, 3, 4, 5, 6)$. According to the linguistic scale and the definition 6, we can see that whatever λ is, the most desirable alternative is x_1, so we chose supplier. The example contains complex fuzzy linguistic information, so that GOWA operator cannot be solved.

5. Conclusion

Although the traditional induced aggregation operator can aggregate the information in numerical form, it still has some shortcomings in processing the fuzzy linguistic induced information. Hence, we have presented fuzzy linguistic induced generalized aggregation operators to deal with the complex fuzzy problems. First, we have introduced the TrFLIGOWA operator by using Trapezoidal order inducing variables in order to deal with the complex fuzzy linguistic information. Meanwhile, we have analyzed some main properties of the TrFLIGOWA operator. We also applied the TrFLIGOWA operator to deal with multiple attribute decision-making problem with fuzzy linguistic information-the food supply. The fact proved that the proposed operator can be used to aggregate the fuzzy linguistic induced information in MAGDM problem. It also enriches the existing approaches to aggregating fuzzy linguistic information. Obviously, it can be seen from the examples of food supply that the proposed operators are practical and effective .

In the future, we will continue to study the properties of the TrFLIGOWA operator, and try to extend the TrFLIGOWA operator to the practical application in many fields.

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