

# Fund Asset Allocation Strategy Analysis based on Systemic Risk

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## Abstract

In this paper, covariance matrix, Monte Carlo model and Markowitz model are established to study stock investment returns, systemic risk and optimal portfolio. Firstly, the similarity of asset allocation strategies of different fund companies is analyzed. It is necessary to establish a model to combine the return of stock investment with systemic risk, and determine the optimal stock portfolio based on it. The model was chosen to be capable of subsequent analysis based on historical data. In this paper, Markowitz model (also known as mean-variance model) is adopted to solve this problem. The model uses historical data to analyze the stocks it invests in, this paper analyzes how to optimize stock investment through known historical effective information, such as mean return and covariance, combining with their own risk preference, acceptable mean return or investment ratio under the condition of investment portfolio, so as to diversify the investment risk of the selected stock portfolio and achieve the ideal goal of maximizing investment utility.

## Keywords

Portfolio Strategy; Maximization of Investment Utility; Value at Risk; Covariance Matrix; Monte Carlo Simulation; Markowitz Model.

## 1. Introduction

In recent years, with the continuous improvement of the degree of reform and opening up, various risks in China's economic operation are gradually exposed and concentrated in the financial field.

Due to the adoption of the relative performance appraisal system, the competitive pressure makes the asset allocation of public funds very complicated, and the shareholding concentration is relatively high, which has become a potential factor affecting systemic risk. Therefore, how to balance the relationship between fund investment returns and systemic risk is worth further exploration.

## 2. Models and Analysis

### 2.1. Terms, Definitions and Symbols

The signs and definitions are generated from *queuing theory*.

- Prob -- namely English Probability, represents the Probability that the fair value loss of an asset or portfolio reaches the upper limit.
- $\Delta P$  -- the amount of loss in value of an asset or portfolio over a particular holding period.
- VaR -- Value at risk at confidence level C, the maximum possible loss.
- C -- given confidence level
- W -- Value of the portfolio at the end of a particular holding period.
- $W^*$  -- the lowest portfolio value under confidence interval C

- R -- Rate of return
- $P_t$  -- asset prices at time T
- $P_0$  -- the asset price at the initial time
- $\gamma_p$  -- Portfolio returns
- $\gamma_i$  -- The yield on stock  $i$
- $\omega_i, \omega_j$  -- The proportion of investment in securities I and j
- $\sigma^2 \gamma_p$  -- Portfolio variance (Total portfolio risk)
- $\text{Cov}(\gamma_i, \gamma_j), \sigma_{i,j}$  -- The covariance between two securities
- V -- covariance matrix
- W -- investment proportion vector of each security in the portfolio
- R -- the return vector of each security
- E ( $\gamma_p$ ) -- A predetermined yield vector

## 2.2. Assumptions

- 1) Suppose the price of the fund is subject to some random process
- 2) Using the computer can simulate the fund price change path in a certain period
- 3) Computer simulations can be repeated n times
- 4) Investors know the probability distribution of return rate of each security and portfolio in advance, and only consider the mean and variance of return rate in the process of investment decision.
- 5) Rational investors assume that investors in the market are risk-averse, and will choose assets with low risk when the return is determined, but with the same risk, they will choose assets with high return.
- 6) Efficient capital markets assume that asset prices can accurately reflect their intrinsic value and investors do not have differences in access to information. There is no friction in the market, it is complete, there are no costs, there are no restrictions on access to the market, and all participants in the market are just price takers.
- 7) The return of assets is uncertain and there are certain risks in investment. The assumption of normality for yields, and you're not allowed to sell short in the market.
- 8) Every security in the capital market is infinitely divisible, and fewer than one share can be bought or sold. Any investor's buying and selling of stocks will not cause changes in the market price, and the market has infinite elasticity of supply.

## 2.3. Basic Data Processing

- 1) The concept of Covariance Matrix and the correlation

Covariance is used in probability theory and statistics to measure the total error of two variables. And variance is a special case of covariance, where two variables are the same.

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Covariance represents the error of the population of two variables, as opposed to variance representing the error of only one variable. If two variables move in the same direction, then the covariance between them is positive. If two variables move in opposite directions, the covariance between them is negative. The covariance  $\text{Cov}(X, Y)$  formula between two real random variables  $X$  and  $Y$  with expected values of  $E[X]$  and  $E[Y]$  is expressed as:

$$\begin{aligned} \text{cov}(X, Y) &= E[(X - E[X])(Y - E[Y])] \\ &= E[XY] - 2E[Y]E[X] + E[X]E[Y] \\ &= E[XY] - E[X]E[Y] \end{aligned}$$

In statistics and probability theory, each element of a covariance matrix is the covariance between the elements of a vector a natural generalization of scalar random variables to higher dimensional random vectors.

Define  $X = (X_1, X_2, \dots, X_N)^T$  as the n-dimensional random variable, called the matrix:

$$C = (c_{ij})_{n \times n} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{pmatrix}$$

the Covariance matrix of n-dimensional random variable X, also denoted as  $D(X)$ , Where

$$c_{ij} = Cov(X_i, X_j), i, j = 1, 2, \dots, n$$

is the covariance of the components  $X_i$  and  $X_j$  of X

In order to quantify the correlation of two things, this question introduces the concept of covariance. For random variables X and Y, the covariance of them is defined as:

$Cov[x, y] = E[(x - \mu) \cdot (y - v)]$ ,  $\mu, v$  are the expected values, Without loss of generality, assuming that  $\mu = 0, v = 0$  and  $Cov[X, Y] = \frac{1}{n-1} X \cdot Y = \frac{1}{n-1} X \cdot Y \cdot \cos\theta$  are independent of each other, the smaller the Angle, the larger the covariance, the more correlated they are, and there is A positive correlation and A negative correlation.

In this case, the covariance matrix is introduced and the covariance between a pair of variables  $X_1, X_2, X_3, \dots, X_{10}$  is expressed in the form of A matrix:

$$\begin{pmatrix} V[X_1] & Cov[X_1, X_2] & Cov[X_1, X_3] \\ Cov[X_2, X_1] & V[X_2] & Cov[X_2, X_3] \\ Cov[X_3, X_1] & Cov[X_3, X_2] & V[X_3] \end{pmatrix}$$

Thus, covariance represents the degree of correlation between two sets of random variables, that is, similarity.

Result:

**Table 1.** Calculation results

|   | A    | B    | C    | D    | E    | F    | G    | H    | I    | J    |
|---|------|------|------|------|------|------|------|------|------|------|
| A | 1.5  | -1.5 | -3.1 | -3.1 | -2.1 | -3.1 | -1.9 | -3.1 | -3.1 | -3.1 |
| B | -1.5 | 1.5  | 1.0  | 5.9  | 1.2  | 4.8  | 3.6  | 9.9  | 6.4  | 1.0  |
| C | -3.1 | 1.0  | 1.5  | 1.6  | 2.1  | -6.5 | 6.8  | 1.4  | 1.6  | 4.1  |
| D | -3.1 | 5.9  | 1.6  | 1.7  | 1.0  | 5.8  | 5.4  | 5.3  | 8.8  | 1.4  |
| E | -2.1 | 1.2  | 2.1  | 1.0  | 1.8  | 7.2  | 4.2  | 1.0  | 1.0  | 1.7  |
| F | -3.1 | 4.8  | -6.5 | 5.8  | 7.2  | 1.7  | -2.3 | 6.7  | 4.5  | 4.3  |
| G | -1.9 | 3.6  | 6.8  | 5.4  | 4.2  | -2.3 | 1.9  | 1.2  | 1.3  | 8.0  |
| H | -3.1 | 9.9  | 1.4  | 5.3  | 1.0  | 6.7  | 1.2  | 1.8  | 1.1  | 1.1  |
| I | -3.1 | 6.4  | 1.6  | 8.8  | 1.0  | 4.5  | 1.3  | 1.1  | 1.7  | 1.3  |
| J | -3.1 | 1.0  | 4.1  | 1.4  | 1.7  | 4.3  | 8.0  | 1.1  | 1.3  | 2.4  |

As can be seen from the figure, the similarity of asset allocation strategies of fund company AC and CA must be equal. The calculated covariance matrix results in the degree of similarity of asset allocation strategies between the ten fund companies, and the larger the value is, the higher the similarity is.

2)The model of the value at risk (VaR)

Value at Risk (VaR) is a financial Risk measurement and control model. VaR represents the maximum possible loss or the maximum possibility of market value change of a certain financial asset (or portfolio) of investors in a specific period of time in the future at a certain confidence level. At present, VaR has become one of the most important indicators to measure market risk. It's expressed by the formula [1]:

$$\begin{aligned}
 \text{Pr ob}(\Delta P > VaR) &= 1 - c \\
 VaR &= E(W) - W^* \\
 \begin{cases} W = W_0(1 + R) \\ W^* = W_0(1 + R^*) \end{cases}
 \end{aligned}$$

Then:

$$VaR = W_0(E(R) - R^*)$$

$W_0$  is the value of the asset portfolio at the beginning of the holding period, and  $R^*$  is the lowest rate of return under confidence degree C. According to  $R^*$ , the possible asset losses of 10 fund companies can be compared under the condition that the holding period is 2020 and confidence degree C =x%.

3)Selection of Monte Carlo

There are three ways [2] to measure VaR below:

**Table 2.** Three ways to measure VaR

|                                  | Historical simulation method | Variance - covariance analysis  | Monte Carlo simulation method  |
|----------------------------------|------------------------------|---|--|
| Computing speed                  | quickly                      | quickly   | Slow unless the portfolio contains a small variety and number of financial instruments |
| Market instability               | The results will be biased   | Results will be biased unless other standard deviations and correlation coefficients are used                                       | The results will be biased unless other distribution parameters are used               |
| Method implementation difficulty | easy                         | easy  | hard   |
| Ability to test other hypotheses | /                            | Other standard deviation and correlation coefficient assumptions can be tested, but other distribution assumptions cannot be tested | /  |

As the fund portfolio contains only 10 funds at most and no other financial instruments are involved, there is no need to collect additional data for the known stocks unless the daily price of the whole year of 2019 is known in the case of trading day or suspension. Due to the moderate amount of data, our team can realize data operation and visualization easily and quickly by using R and Python. In addition, by adjusting the distribution parameters of the model, the distortion of the model caused by market risks can be avoided. Therefore, the Monte

Carlo simulation method is selected to establish the VaR prediction model of investment fund portfolio.

4) Geometric Brownian motion

Geometric Brownian motion is the most commonly used model in the process of financial asset price random movement. Here, we choose geometric Brownian motion as the random model of fund yield change for Monte Carlo simulation analysis, and the stochastic equation of geometric Brownian motion is derived by ITO's lemma:

$$dS_t = \mu_t S_t dt + \sigma_t S_t dz$$

$dz$  is the normal distribution with  $dz$  mean of 0 and  $dz$  variance of  $dt$ , and the parameters  $\mu_t$  and  $\sigma_t$  represent the instantaneous drift rate and volatility respectively, which can be approximated as constants. It is difficult and unnecessary to obtain real-time price data when describing fund price changes, so it is more convenient to calculate by converting the above formula into discrete form of time:

$$\Delta S_{t+1} = S_t(\mu_t \Delta t + \sigma_t \varepsilon_t \Delta t)$$

$t = \frac{T-t}{n}$ , the present moment is  $t$ , and the adjacent maturity moment of the next holding period is  $T$ .  $n$  represents the number of segments divided into 2020 by time. Given the daily price movements of 57 funds, equation  $\Delta t = 1$  above is obtained

$$S_{t+1} = S_t + S_t(\mu_t \Delta t + \sigma_t \varepsilon_t \sqrt{\Delta t})$$

At time  $t$ ,  $S_t$  is calculated and corresponding parameters  $\mu_t$  and  $\varepsilon_t$  ( $t=1,2,\dots,n$ ) are estimated. Substitute  $\varepsilon_t$  into the formula to obtain  $S_{t+2}$ , and then  $S_{t+n}$  can be obtained. Repeating this process for several times can simulate the price distribution in the future period, and the VaR of the fund portfolio can be obtained according to the confidence degree.

5) Solution of model

By analyzing the strategies of 10 fund companies adopting fund portfolio investment in 2019, the weight of each fund should be considered when using Monte Carlo method to simulate VaR in 2020.

Suppose  $r_i^*$  represents the return rate of fund portfolio  $i$  when taking  $w^*$  at the confidence level  $c=95\%$ ,  $P_i$  represents the final value of the  $i$ th investment strategy at the end of 2019,  $x_i$  represents the weight of the  $i$ th fund in the total investment assets in an investment strategy, ( $i=1,2,\dots,10$ ) and B converts the formula to

$$\begin{aligned} VaR(R_i) &= E(W) - W^* \\ &= W_0 \sum_{i=1}^n P_i [E(R_i) - r_i^*] \end{aligned}$$

It can be seen from the above equation that VaR of fund portfolio is a homogeneous linear function of fund weight  $x_i$ .

The generation of random variables and the calculation of VaR are achieved with the help of R language programming. Finally, the simulated value of VaR of the investment strategy of 10 fund companies in 2020 is obtained. In order to avoid the error caused by accidental factors in the simulation process, the team also adopted the historical simulation method to test the simulation results of Monte Carlo method, and the data are as follows:

**Table 3.** Simulation result verification data (a)

|                       | A          | B          | C         | D          | E          |
|-----------------------|------------|------------|-----------|------------|------------|
| VaR(\$)(MentoCarlo)   | 0.04048175 | 0.04079075 | 0.0519927 | 0.04119227 | 0.03537364 |
| VaR(\$)( Historical ) | 0.04503662 | 0.04984613 | 0.0513659 | 0.05064735 | 0.04529989 |

(Continue to the above-mentioned)

**Table 4.** Simulation result verification data (b)

|                | F          | G          | H          | I          | J         |
|----------------|------------|------------|------------|------------|-----------|
| (Mento Carlo)  | 0.03839754 | 0.03943661 | 0.03803304 | 0.03589286 | 0.0336301 |
| ( Historical ) | 0.0545336  | 0.05078294 | 0.04875053 | 0.04880778 | 0.0332831 |

Through comparison, we find that although the VaR of each company calculated by the two methods is different, the difference is less than 0.01, indicating that the VaR obtained by monte Carlo simulation method is basically credible. The value-at-risk ranking of each fund company at 95% confidence level in 2020 is:

$$C (0.0519927) > D (0.04119227) > B (0.04079075) > A (0.04048175) > G (0.03943661) > F (0.03839754) > H (0.03803304) > I (0.03589286) > E (0.03537364) > J (0.0336301)$$

## 2.4. The Final Model Used--Markowitz

### 1) Choice of Markowitz model

In 1952, Markowitz put forward the mean-variance model for the first time, using the mean and variance to represent the return and risk of assets respectively. The problem of asset allocation was transformed into a quadratic programming problem, that is, seeking the portfolio with the least risk under the given return, or seeking the portfolio with the maximum return under the given risk.

Established the famous Markowitz portfolio theory. According to the model portfolio efficient frontier curve, between the risks and benefits, the risk of investors according to their own preferences, to seek the optimal portfolio, at the same time pointed out that due to the income correlation between individual stocks, investors can be dispersed by constructing portfolio due to uncertainties, the stocks of systemic risk, in turn, to reduce the risk level of the overall portfolio.

According to the literature, Markowitz asset portfolio theory has been of significant application value in China's stock market [3].

### 2) data processing

The covariance of the 57 stocks is:

If investors make investment decisions based on the expected return rate, that is, the mean value, when the actual return is lower than the expected return rate, investors will take certain risks. There is usually a certain deviation between the two. The larger the deviation is, the more dispersed the actual return rate of the asset is, and the greater the risk investors face when

investing. Therefore, in mathematics, variance is used to measure the deviation between the actual rate of return and the expected rate of return to reflect the size of investment risk.

**Table 5.** The covariance of the 57 stocks

|                 | <i>stock 1</i> | <i>stock 2</i> | <i>stock 3</i> | ... | <i>stock 55</i> | <i>stock 56</i> | <i>stock 57</i> |
|-----------------|----------------|----------------|----------------|-----|-----------------|-----------------|-----------------|
| <i>stock 1</i>  | 0.191443       | 0.070345       | 0.054187       | ... | 0.063769        | 0.003741        | 0.030781        |
| <i>stock 2</i>  | 0.070345       | 0.140458       | 0.054178       | ... | 0.066858        | -0.008846       | 0.034894        |
| <i>stock 3</i>  | 0.054187       | 0.054178       | 0.119327       | ... | 0.048260        | -0.009190       | 0.029331        |
| <i>stock 4</i>  | 0.069979       | 0.039484       | 0.038093       | ... | 0.051449        | -0.017697       | 0.013238        |
| <i>stock 5</i>  | 0.109130       | 0.063180       | 0.057653       | ... | 0.101473        | -0.005926       | 0.038993        |
| ...             | ...            | ...            | ...            | ... | ...             | ...             | ...             |
| ...             | ...            | ...            | ...            | ... | ...             | ...             | ...             |
| <i>stock 55</i> | 0.063769       | 0.066858       | 0.048260       | ... | 0.208687        | -0.006712       | 0.045846        |
| <i>stock 56</i> | 0.003741       | -0.008846      | -0.009190      | ... | -0.006712       | 0.143086        | -0.001737       |
| <i>stock 57</i> | 0.030781       | 0.034894       | 0.029331       | ... | 0.045846        | -0.001737       | 0.092861        |

Variance is used to measure the risk of the portfolio, so the variance of the portfolio is:

$$\begin{aligned} \sigma_p^2 &= \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + \dots + \omega_n^2 \sigma_n^2 + \dots + \omega_1 \omega_2 \sigma_{1,2} + \omega_1 \omega_3 \sigma_{1,3} + \dots + \omega_1 \omega_n \sigma_{1,n} + \dots \\ &= (\omega_1, \omega_2, \dots, \omega_n) \begin{pmatrix} \sigma_1^2 & \sigma_{1,2} & \dots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & \dots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \dots & \sigma_n^2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix} \\ &= W^T V W \end{aligned}$$

The expected annualized return of the portfolio is calculated as follows:

**Table 6.** The expected annualized return of the portfolio

|             | <i>Expected annualized earnings</i> |
|-------------|-------------------------------------|
| <i>ST A</i> | 0.4520000512730448                  |
| <i>ST B</i> | 0.5848858576453517                  |
| <i>ST C</i> | 0.8105981981535033                  |
| <i>ST D</i> | 0.4533770087729643                  |
| <i>ST E</i> | 0.5529836810727062                  |
| <i>ST F</i> | 0.6805573691954282                  |
| <i>ST G</i> | 0.42066966960415647                 |
| <i>ST H</i> | 0.610231254615895                   |
| <i>ST I</i> | 0.5314102879886131                  |
| <i>ST J</i> | 0.5348774983101772                  |

The expected portfolio variance is calculated as follows:

**Table 7.** The expected portfolio variance

|             | <i>Expected portfolio variance</i> |
|-------------|------------------------------------|
| <i>ST A</i> | 0.05545597934551573                |
| <i>ST B</i> | 0.05123631329886684                |
| <i>ST C</i> | 0.07766063722127652                |
| <i>ST D</i> | 0.046314692021869275               |
| <i>ST E</i> | 0.044282730059689475               |
| <i>ST F</i> | 0.027449792428921344               |
| <i>ST G</i> | 0.0428563606240395                 |
| <i>ST H</i> | 0.05513597041074933                |
| <i>ST I</i> | 0.0439821484326993                 |
| <i>ST J</i> | 0.047463880760862116               |

The expected portfolio standard deviation is calculated as follows:

**Table 8.** The expected portfolio standard deviation

|      | <i>Standard deviation of expected portfolio</i> |
|------|---|
| ST A | 0.23549093261846776                             |
| ST B | 0.22635439756909262                             |
| ST C | 0.27867658175971033                             |
| ST D | 0.21520848501364734                             |
| ST E | 0.2104346218180114                              |
| ST F | 0.16567978883654258                             |
| ST G | 0.20701777852165137                             |
| ST H | 0.23481049893637493                             |
| ST I | 0.2097192133131805                              |
| ST J | 0.21786206820110315                             |

3) The establishment of Markowitz model

To use Markowitz model to find the optimal portfolio, first:

Establish the objective function:

$$\min \sigma^2(\gamma_p) = \sum \sum \omega_i \omega_j \text{Cov}(\gamma_i, \gamma_j)$$

$$E(\gamma_p) = \sum \omega_i \gamma_i$$

Set limits:

$$1 = \sum \omega_i, \quad \omega_i > 0 \quad (\text{Short selling of stocks is not allowed})$$

According to Lagrange method, the above quadratic linear programming problem is solved, and the relationship between the investment proportion of each security in the portfolio and the expected return rate is obtained as follows:

$$W = g + hE_p$$

$$g = \frac{1}{D} (BV^{-1}e - AV^{-1}R)$$

$$h = \frac{1}{D} (CV^{-1}R - AV^{-1}e)$$

Where, A, B, C and D are all constants, and the values are affected by the return and variance of each asset in the portfolio, and the specific value is  $A = e^T V^{-1} R = R^T V^{-1} e$ ,  $B = R^T V^{-1} R$ ,  $C = e^T V^{-1} e$ ,  $D = BC - A^2$ .

The effective frontier curve of the portfolio reflects the minimum variance that the portfolio can achieve under a given expected rate of return. For rational investors, the frontier portfolio can maximize its investment utility. The equation of the effective frontier curve of the portfolio is as follows:

$$\sigma_p^2 = \frac{C}{D} (E_p - \frac{A}{C})^2 + \frac{1}{C}$$

It can be seen from the equation that  $\sigma_p^2$  is an increasing function of  $E_p$ , which means that the risk increases with the increase of income. In  $(\sigma_p^2, E_p)$ , the effective frontier curve is  $\sigma_p^2$  hyperbola. On this curve, the investor has the highest return under the given risk, and the portfolio below the curve is suboptimal for the investor. Investors can choose the portfolio with the maximum utility from the effective frontier curve according to their personal risk preference [4].

4) Solution of Markowitz model

To solve the Markowitz model, generally follow the following steps:

The first step is to determine the feasible investment set of risk assets and further find out the risk and return opportunities faced by investors, which is to determine the effective frontier of the corresponding risk assets.



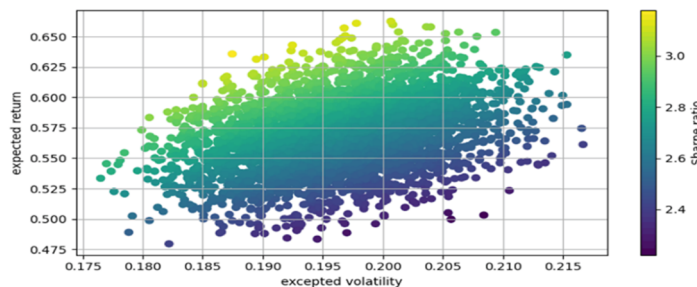
The second step is to find the allocation ratio of risky assets with the maximum Sharpe ratio on the efficient frontier, that is, to find the tangential portfolio.

The third step is to determine the investment proportion of the portfolio in risk-free assets and N kinds of risk assets after the given risk aversion coefficient A of investors, so as to construct the optimal portfolio [5]

Finally, according to the model, the standard deviation of portfolio return rate is the abscissa and the expected return rate of portfolio is the ordinate, and the boundary of all feasible set combinations forms a curve, which is called the frontier curve. The meaning of effective frontier indicates that there is a nonlinear form between return and risk, and each point is the optimal set of portfolios, representing the investment decisions made by all rational investors.

Next, the model is solved according to the above steps:

Monte Carlo method is used to simulate a large number of random portfolios, and the expected returns and variances of random portfolios are recorded (here the risk-free interest rate is given as 4%):



**Fig 1.** The Markowitz model

Portfolio Optimization 1: Sharp is the biggest.

Given the risk-free rate  $\sigma_f$  we define the Sharpe ratio (or return to volatility ratio) of portfolio P as:

$$S = \frac{E(Y_p) - \gamma_p}{\sigma_f}$$

When the capital allocation line slope and the effective frontier tangent, risk portfolio reached the maximum sharpe ratio, point of contact for a portfolio on the efficient frontier of the optimal capital allocation, the distribution of the capital market and achieve equilibrium, all valid combination is different risk preferences of investors in the risk-free asset and the result of the free allocation between the market portfolio.

The optimal combination weight vector is calculated when Sharpe is at its maximum:

**Table 9.** The optimal combination weight vector is calculated when Sharpe is at its maximum

|       |       |    |    |       |       |    |       |    |    |    |    |
|-------|-------|----|----|-------|-------|----|-------|----|----|----|----|
| 0.134 | 0.    | 0. | 0. | 0.034 | 0.    | 0. | 0.    | 0. | 0. | 0. | 0. |
| 0.    | 0.    | 0. | 0. | 0.    | 0.001 | 0. | 0.    | 0. | 0. | 0. | 0. |
| 0.    | 0.    | 0. | 0. | 0.    | 0.    | 0. | 0.    | 0. | 0. | 0. | 0. |
| 0.52  | 0.239 | 0. | 0. | 0.    | 0.    | 0. | 0.    | 0. | 0. | 0. | 0. |
| 0.    | 0.    | 0. | 0. | 0.    | 0.    | 0. | 0.072 | 0. | 0. | 0. | 0. |

When Sharpe is at its maximum, the expected return rate, expected volatility and Sharpe index of portfolios are:

$$[ 0.494 \quad 0.033 \quad 15.032 ]$$

Portfolio optimization 2: Minimum variance

The optimal combination weight vector with the smallest variance:

**Table 10.** The optimal combination weight vector with the smallest variance

$$[0.136 \ 0. \ 0. \ 0. \ 0.029 \ 0.001 \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \\ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \\ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \\ 0.517 \ 0.245 \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \\ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0. \ 0.072 \ 0. \ ]$$

The expected return rate, expected volatility and optimal Sharpe index with the smallest variance are:

$$[ 0.489 \ 0.033 \ 14.954 ]$$

Summarized as:

Sharpe's optimal stock portfolio strategy at the peak is:

**Table 11.** Sharpe's optimal stock portfolio strategy at the peak

|                 | <i>proportion (%)</i> |
|-----------------|-----------------------|
| <i>stock1</i>   | 13.4                  |
| <i>stock 5</i>  | 3.4                   |
| <i>stock 18</i> | 0.1                   |
| <i>stock 37</i> | 52                    |
| <i>stock 38</i> | 23.9                  |
| <i>stock 56</i> | 7.2                   |

The optimal stock portfolio strategy with the smallest variance is:

**Table 12.** The optimal stock portfolio strategy with the smallest variance

|                 | <i>proportion (%)</i> |
|-----------------|-----------------------|
| <i>stock 1</i>  | 13.6                  |
| <i>stock 5</i>  | 2.9                   |
| <i>stock 6</i>  | 0.1                   |
| <i>stock 37</i> | 51.7                  |
| <i>stock 38</i> | 24.5                  |
| <i>stock 56</i> | 7.2                   |

### 3. Conclusion

#### 3.1. Strength of the Model

1. VaR is chosen to measure the value at risk of fund companies, to link historical volatility and correlation of specific investment strategies, and to predict future price risks.
2. Monte Carlo simulation method is used to set various actual parameters according to the historical data for simulation, and accurate results are given.
3. Markowitz effective portfolio is significantly better than random simple equal-weight portfolio, with small risk and large return, which can be measured by variation coefficient (return/standard deviation). If the variation coefficient of The Markowitz efficient portfolio is higher than that of the random simple equal-weight portfolio, it indicates that the return rate of the efficient portfolio is higher than that of the random portfolio under the given risk. Or on a given return, the efficient portfolio is less risky than the random portfolio[3].
4. The effective portfolio of Markowitz is small and concentrated, so investors can concentrate on the stocks with a relatively large proportion of investment, instead of dispersing management resources like equal-weight portfolio. Because Markowitz portfolio adopts optimization method to determine the investment proportion of various securities, focusing on reducing the correlation between various yields, while excluding some low-yield and high-risk securities [3].

### 3.2. Weakness of the Model

1. Fully dynamic planning is not fully considered. Although historical VaR can help us to exceed the rated loss every time in a year, it cannot inform the possibility of loss occurring in the original trading strategy at a certain point.
2. Monte Carlo simulation is carried out according to geometric Brownian model, which has many parameters, resulting in redundant parameters and large errors.
3. Markowitz's portfolio theory is based on a series of strict assumptions. However, due to the possibility of market failure, a fully efficient stock market is an ideal state, while only a secondary efficient market exists in reality [4].
4. On the assumption that investors only pay attention to the mean and variance of return rate, Markowitz's portfolio theory is completely accurate. However, the biggest problem it faces is the large amount of calculation, which affects its practical application [4].
5. The reality of the securities market, securities gains has very strong timeliness, which requires the securities investment decision method is also with time-varying characteristics, and the Markowitz mean - variance estimate of the parameters in the model, request the sample length is long enough, and the sample through long will cause model parameter is not fully reflect the latest changes of the stock returns, Therefore, its timeliness is poor [3].

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