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Research on Supply Chain Network Equilibrium Considering Risk Function

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Abstract

Economic globalization continues to expand the scope of the supply chain network structure, while increasing its own complexity, as well as the uncertainty of the network operating environment and the fragility of the operating system. An emergency on a single node or line in the supply chain network usually affects other nodes in the supply chain and brings significant risks to the enterprise. The impact of other nodes can cause the entire supply chain network to collapse, especially if the production and operation of a single-node enterprise in the supply chain may be interrupted or malfunctioned, especially in the event of an emergency. It also threatens development greatly, affecting the production and livelihoods of enterprises in the supply chain and people's lives, and has a major negative impact on social and economic development. These emergencies continue to affect the supply chain network, and the originally fragile companies face greater risks. The research content of this paper is: Firstly, use the super network model to establish a supply chain super network model considering the risk function, including multiple raw material suppliers, multiple manufacturers, multiple retailers, multiple demand markets, and maximize their respective profits. , The minimum risk is the goal, the super network model is established separately to analyze the behavioral decisions and goals of enterprises at all levels; the method of variational inequality is used to find the system equilibrium conditions, and the projection gradient algorithm is used to find the equilibrium solution in the last step.

Keywords

Supply Chain Network; Risk; Profit Maximization.

1. Introduction

In reality, the supply chain is faced with many risks, which have a significant impact. For example: the new crown pneumonia epidemic, an emergency public health event in 2019. The "2021 World Trade Report" pointed out that the health and economic crisis triggered by the epidemic is a large-scale test of the strength of the world trading system. It has had an unprecedented impact on global supply chains and trade relations between countries. In 2020, the value of global trade in goods and services in nominal U.S. dollars fell by 9.6%, and global GDP fell by 3.3%. This was the worst recession since World War II. The impact of the epidemic on different industries is not the same. The 2021 Maritime Transport Review issued by the United Nations Conference on Trade and Development pointed out that maritime trade has shrunk by 3.8% in 2020, but it has since rebounded and is estimated to increase by 4.3% this year. The report shows that the medium-term prospects are still optimistic, but they are also facing more and more risks and uncertainties. For example, the global supply chain is facing unprecedented pressure, freight rates have soared, import prices have increased prices, and crews have been stranded due to the new crown pandemic. Labor shortage caused by the sea, etc. In general, this epidemic has a relatively important impact on the international supply chain.

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As the types of risks increase, their influence increases, and their complexity increases, the existing research is relatively inadequate to deal with the impact of risks. How to improve the overall resistance of the supply chain and enhance the coordination ability of the supply chain is particularly important.

Scholars at home and abroad have done a lot of research on supply chain network. Nagurney[1] et al. studied the relationship between supply chain network members and their Decision-making behavior under the condition of a single commodity with a certain demand. Zhang Yang Hammond D[2] et al. studied the closed-loop supply chain equilibrium with transaction oligopoly as an influencing factor. Chen[3] et al. studied the supply chain network equilibrium problem with capacity constraints (SCNE-C). Jiang [4] et al. studied a new model of green supply chain oligopoly competition for a single product, taking environmental factors into consideration. Wenliang Zhou [5] et al. studied a multi-period, multi-commodity flow supply chain network equilibrium problem with a postponement strategy in a supply chain management model. Shiqin Xu[6] studied a supply chain network equilibrium model, in which e-commerce is carried out in the presence of B2B (business to business) and B2C (business to consumer) transactions, and multi-period Decision-making and multi-criteria Decision-making are integrated.

2. Supply Chain Network Considering Risk Function

2.1. Super Network Model Structure Considering Risk Function

This chapter studies the equilibrium state of each raw material supplier, manufacturer, retailer, and consumer market in the supply chain network considering the risk function. In this process, each enterprise seeks to maximize benefits while minimizing risks.

Before analyzing the optimization status of decision makers at various levels, for the convenience of research, it is assumed that: raw material suppliers, manufacturers, and retailers compete with each other at the same level and cooperate with each other; the enterprise supply chain is seamless, that is, inventory costs and Time cost; the consumer pays once after receiving the finished product; the consumer does not designate the raw material supplier to treat the manufacturing process as a person's overall activity, and does not consider the sub-activity of each process.

2.2. Behavioral Decision-making and Target Analysis at All Levels

In order to concisely describe each level of decision-makers and their goals, replace the numbers involved in the Decision-making process with the following symbols:

M : The total number of raw material suppliers,

N : The total number of manufacturers,

K : Total number of retailers

J : The total number of consumers,

q_{mn} : The quantity of raw materials sold by the raw material supplier to the manufacturer,

q_{nk} : The number of products sold by the manufacturer to the retailer,

q_{kj} : The number of retailers sold to consumers,

p_{mn} : The price that the raw material supplier sells to the manufacturer,

p_{nk} : The price at which the manufacturer sells the product to the retailer,

P_{kj} : The price at which the retailer sells the product to the consumer,

h_{mn} : The level of the relationship between the raw material supplier and the manufacturer,

h_{nk} : Manufacturer Retailer The relationship level at which transactions are conducted.

Raw material supplier behavior and target analysisa person's overall activity, and does not consider the sub-activity of each process.

2.2.1. Raw Material Supplier Behavior and Target Analysis

(1) Profit maximization representation

In this model, raw material suppliers are mainly responsible for purchasing and collecting the most original materials. In this process, pay the corresponding purchase cost, which is represented by f_m. There are transaction fees when dealing with manufacturers c_{1mn}. In order to make the transaction proceed smoothly and help the next transaction occur, a certain level of relationship must be established, and the resulting cost is represented by the economic cost function v_{mn}. For the raw material supplier:

The purchase cost is

$$f_m = f_m(Q_1), \forall m \tag{1}$$

The transaction cost function is a continuously differentiable convex function about the transaction volume q_{mn} and the relationship level h_{mn}

$$c_{1mn} = c_{1mn}(q_{mn}, h_{mn}), \quad \forall q_{mn} \geqslant 0, \quad 0 \leqslant h_{mn} \leqslant 1 \tag{2}$$

The economic cost function is

$$v_{mn} = v_{mn}(h_{mn}) \tag{3}$$

The maximum profit of the raw material supplier available from the above is

$$Max\ Z_m = \sum_{n=1}^{N} p_{mn} q_{mn} - f_{m(Q_1)} - \sum_{n=1}^{N} c_{1mn}(q_{mn}, h_{mn}) - \sum_{n=1}^{N} v_{1mn}$$
$$\forall q_{mn} \geqslant 0, \quad 0 \leqslant h_{mn} \leqslant 1 \tag{4}$$

(2) Representation of minimum risk

When raw material suppliers conduct transactions with manufacturers, they must also consider certain emergency situations, such as product transportation losses, which bring certain risks to the raw material suppliers. Here, the relationship level h is used to measure this type of impact. The greater the level of the established relationship h_{mn}, the more stable the two parties representing the cooperation, and the smaller the risk assumed. We use r_{mn} to represent the risk function

$$r_{mn} = r_{mn}(q_{mn}, h_{mn}), \quad \forall m, n \tag{5}$$

The minimum risk borne by the raw material supplier is

$$Min\ \ R_{mn} = \sum_{n=1}^{N} r_{mn}(q_{mn}, h_{mn}) \tag{6}$$

$$\forall m, n \ \ q_{mn} \geqslant 0, 0 \leqslant h_{mn} \leqslant 1$$

(3) Analysis of optimization status of raw material suppliers

The raw material supplier m pays different attention to risk. Here, the weight α_{mn} is used to indicate the degree of importance the raw material supplier attaches to the risk. According to formulas (4) and (6), an objective function of the raw material supplier m can be obtained

$$Max\ Z_m - \beta_m r_m = \sum_{n=1}^{N} p_{mn} q_{mn} - f_{m(Q_1)} - \sum_{n=1}^{N} c_{1mn}(q_{mn}, h_{mn}) - \sum_{n=1}^{N} v_{1mn} - \alpha_m \sum_{n=1}^{N} r_{mn}(q_{mn}, h_{mn}) \tag{7}$$

$$\forall m,n \ \ q_{mn} \geqslant 0,\ 0 \leqslant h_{mn} \leqslant 1$$

Assuming that the above functions are all continuous and convex, according to the theory of variational inequality, (7) can be reduced to the following variational inequality

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial f_m(Q_1^*)}{\partial q_{mn}} + \frac{\partial c_{1mn}(q_{mn}^*, h_{mn}^*)}{\partial q_{mn}} - p_{mn}^* + \alpha_m \frac{\partial r_{mn}(q_{mn}^*, h_{mn}^*)}{\partial q_{mn}}\right) \times (q_{mn} - q_{mn}^*) + \left(\frac{\partial c_{1mn}(q_{mn}^*, h_{mn}^*)}{\partial h_{mn}} - \frac{\partial v_{1mn}(h_{mn}^*)}{\partial h_{mn}} + \alpha_m \frac{\partial r_{mn}(q_{mn}^*, h_{mn}^*)}{\partial h_{mn}}\right) \times (h_{mn} - h_{mn}^*) \geqslant 0 \tag{8}$$

2.2.2. Manufacturer's Behavior and Target Analysis

(1) Representation of profit maximization

It can be seen from the previous section that transaction costs and economic costs are required for transactions, denoted as c_{2mn} , v_{mn} . respectively. In the same way, manufacturers and retailers also have to pay transaction costs and economic costs, denoted as c_{2nk} and v_{2nk}.

$$c_{2mn} = c_{2mn}(q_{mn}, h_{mn}),\ \ \forall m,n \tag{9}$$

$$c_{2nk} = c_{2nk}(q_{nk}, h_{nk}),\ \ \forall n,k \tag{10}$$

$$v_{2mn} = v_{2mn}(h_{mn}),\ \ \forall m,n \tag{11}$$

$$v_{2nk} = v_{2nk}(h_{nk}),\ \ \forall n,k \tag{12}$$

The functions defined above are all continuous differentiable convex functions. The manufacturer's maximum profit can be expressed as

$$Max\ Z_n = \sum_{k=1}^{K} p_{nk} q_{nk} - \sum_{k=1}^{K} c_{2nk}(q_{nk}, h_{nk}) - \sum_{m=1}^{M} p_{mn} q_{mn} - \sum_{m=1}^{M} c_{2mn}(q_{mn}, h_{mn}) - \sum_{m=1}^{M} v_{2mn}(h_{mn}) - \sum_{k=1}^{K} v_{2nk}(h_{nk}) \tag{13}$$

$$\sum_{m=1}^{M} q_{mn} \geqslant \sum_{k=1}^{K} q_{nk}, \forall m,n,k \tag{14}$$

$\forall m,n,k$, $q_{mn} \geqslant 0, q_{nk} \geqslant 0, 0 \leqslant h_{mn} \leqslant 1, 0 \leqslant h_{nk} \leqslant 1$

(2) Representation of minimum risk

When manufacturers trade with raw material suppliers and retailers, they also face certain risks. Here, r_{2mn} and r_{2nk} are used to denote:

$$r_{2mn} = r_{2mn}(q_{mn}, h_{mn}), \quad \forall m, n \tag{15}$$

$$r_{2nk} = r_{2nk}(q_{nk}, h_{nk}), \quad \forall n, k \tag{16}$$

Then the manufacturer's minimum risk is expressed as

$$Min\ R_n = \sum_{m=1}^{M} r_{2mn}(q_{mn}, h_{mn}) + \sum_{k=1}^{K} r_{2nk}(q_{nk}, h_{nk}) \tag{17}$$

(3) Analysis of manufacturer's optimization status

Here a weight value α_n is set for the risk function. According to formulas (13) and (16), a manufacturer's objective function can be obtained:

$$\begin{aligned} Max\ Z_n - \alpha_n R_n = & \sum_{k=1}^{K} p_{nk} q_{nk} - \sum_{k=1}^{K} c_{2nk}(q_{nk}, h_{nk}) - \sum_{m=1}^{M} p_{mn} q_{mn} - \sum_{m=1}^{M} c_{2mn}(q_{mn}, h_{mn}) - \\ & - \alpha_n \left(\sum_{m=1}^{M} r_{2mn}(q_{mn}, h_{mn}) + \sum_{k=1}^{K} r_{2nk}(q_{nk}, h_{nk}) \right) - \sum_{m=1}^{M} v_{2mn}(h_{mn}) - \\ & \sum_{k=1}^{K} v_{2nk}(h_{nk}) \end{aligned} \tag{18}$$

$\forall m, n, k, \quad q_{mn} \geqslant 0, q_{nk} \geqslant 0, 0 \leqslant h_{mn} \leqslant 1, 0 \leqslant h_{nk} \leqslant 1$

Assuming that the above functions are all continuous and convex, according to the theory of variational inequality, (7) can be reduced to the following variational inequality

$$\begin{aligned} & \sum_{m=1}^{M} \sum_{n=1}^{N} \left(\frac{\partial c_{2mn}(q_{mn}^*, h_{mn}^*)}{\partial q_{mn}} + \frac{\partial r_{2mn}(q_{mn}^*, h_{mn}^*)}{\partial q_{mn}} + p_{mn}^* - \zeta_{1n} \right) \times (q_{mn} - q_{mn}^*) + \\ & \sum_{m=1}^{M} \sum_{n=1}^{N} \left(\frac{\partial c_{2mn}(q_{mn}^*, h_{mn}^*)}{\partial h_{mn}} + \frac{\partial r_{2mn}(q_{mn}^*, h_{mn}^*)}{\partial h_{mn}} + \frac{\partial v_{2mn}(h_{mn}^*)}{\partial h_{mn}} \right) \times (h_{mn} - h_{mn}^*) + \\ & \sum_{n=1}^{N} \sum_{k=1}^{K} \left(\frac{\partial c_{2nk}(q_{nk}^*, h_{nk}^*)}{\partial q_{nk}} + \frac{\partial r_{2nk}(q_{nk}^*, h_{nk}^*)}{\partial q_{nk}} - p_{nk}^* + \zeta_{1n}^* \right) \times (q_{nk} - q_{nk}^*) + \\ & \sum_{n=1}^{N} \sum_{k=1}^{K} \left(\frac{\partial c_{2nk}(q_{nk}^*, h_{nk}^*)}{\partial h_{nk}} + \frac{\partial r_{2nk}(q_{nk}^*, h_{nk}^*)}{\partial h_{nk}} - \frac{\partial v_{2nk}(h_{nk})}{\partial h_{nk}} \right) \times (h_{nk} - h_{nk}^*) + \\ & \sum_{n=1}^{N} \left(\sum_{m=1}^{M} q_{mn}^* - \sum_{k=1}^{K} q_{nk}^* \right) \times (\zeta_{1n} - \zeta_{1n}^*) \geqslant 0 \end{aligned} \tag{19}$$

2.2.3. Retailer's Behavior and Target Analysis

(1) Representation of profit maximization

According to the previous assumptions, inventory costs and time costs are not considered. The cost of a retailer's transaction with a manufacturer and consumer is c_{3nk} and c_{kj} , which are both continuous convex functions,

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$$c_{3nk}=c_{3nk}(q_{nk},h_{nk}),\quad \forall n,k \tag{20}$$

$$c_{kj}=c_{kj}(q_{kj}),\quad \forall k,j \tag{21}$$

The economic cost of the relationship level h_{nk} between the retailer and the manufacturer is expressed as v_{3nk},

$$v_{3nk}=v_{3nk}(k_{nk}),\quad \forall n,k \tag{22}$$

According to the above definition, the retailer's maximum profit is expressed as

$$Max\ Z_k=\sum_{j=1}^{J}p_{kj}q_{kj}-\sum_{n=1}^{N}p_{nk}q_{nk}-\sum_{n=1}^{N}c_{3nk}(q_{nk},h_{nk})\ -\sum_{j=1}^{J}c_{kj}(q_{kj})-\sum_{n=1}^{N}v_{3nk}(k_{nk}) \tag{23}$$

$$\sum_{n=1}^{N}q_{nk}=\sum_{j=1}^{J}q_{kj},\forall n,k,j \tag{24}$$

$$\forall n,k,j\quad q_{nk}\geqslant 0\,,q_{kj}\geqslant 0\,,0\leqslant k_{nk}\leqslant 1$$

(2) Representation of minimum risk

Retailers and manufacturers also have certain risks when dealing with them, denoted as r_{3nk}. The retailer delivers the product directly to the consumer, and there is a one-to-one correspondence between the two, so there is no risk, or in other words, the risk is small and can be ignored.

$$r_{3nk}=r_{3nk}(q_{nk},h_{nk}),\quad \forall n,k \tag{25}$$

Then the minimum risk of the retailer is

$$Min\ R_k=\sum_{n=1}^{N}r_{3nk}(q_{nk},h_{nk}) \tag{26}$$

$$\forall n,k\ \ q_{nk}\geqslant 0\,,0\leqslant h_{nk}\leqslant 1$$

(3) Analysis of retailer optimization status

Assuming that the retailer's weight for risk is α_k, according to formulas (22) and (25), a retailer's objective function can be obtained,

$$\begin{aligned}Max\ Z_k-\alpha_k R_k=&\sum_{j=1}^{J}p_{kj}q_{kj}-\sum_{n=1}^{N}p_{nk}q_{nk}-\sum_{n=1}^{N}c_{3nk}(q_{nk},h_{nk})\ -\sum_{j=1}^{J}c_{kj}(q_{kj})-\\&\sum_{n=1}^{N}v_{3nk}(k_{nk})-\alpha_k\sum_{n=1}^{N}r_{3nk}(q_{nk},h_{nk})\end{aligned} \tag{27}$$

Assuming that the above functions are all continuous and convex, according to the theory of variational inequality, (7) can be reduced to the following variational inequality

$$\begin{aligned}&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}+\alpha_k\frac{\partial r_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}+p_{nk}^*-\zeta_{2k}^*\right)\times(q_{nk}-q_{nk}^*)+\\&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}+\alpha_k\frac{\partial r_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}+\frac{\partial v_{3nk}(k_{nk}^*)}{\partial h_{nk}}\right)\times(h_{nk}-h_{nk}^*)+\\&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\frac{\partial c_{kj}(q_{kj}^*)}{\partial q_{kj}}-p_{kj}^*+\zeta_{2k}^*\right)\times(q_{kj}-q_{kj}^*)+\sum_{k=1}^{K}\left(\sum_{n=1}^{N}q_{nk}^*-\sum_{j=1}^{J}q_{kj}^*\right)\times(\zeta_{2k}-\zeta_{2k}^*)\geqslant 0\end{aligned} \tag{28}$$

ζ_{2k} is the Lagrangian coefficient related to the constraint (23), and all ζ_{2k} forms the vector matrix ζ_2.

2.2.4. Consumer Behavior Analysis

In the supply chain network, the consumer is at the bottom, and his purchase behavior of retailers is the promoter of the entire supply chain.

The degree of satisfaction that consumers buy goods that can bring them both is called utility. Let U_{kj} denote the consumer's satisfaction level after purchasing the product from the retailer. The quantity and price of purchased goods will have a direct impact on consumer utility. $U_{kj}(p_{kj},q_{kj})=\beta_0+\beta_1(p_{kj}q_{kj})$ where $\beta_1(\beta_1\geqslant 1)$ represents the consumer's sensitivity to the quantity and price of the product, and β_0 is a non-negative number, which summarizes the utility of consumers to other characteristics of the retailer.

Let d_j denote the consumer market's demand function for retailer's products, the demand function of normal goods is a monotonically decreasing function of price, which can be expressed as

$$d_j=d_j(p),\forall j \tag{29}$$

Consumers will incur certain expenses in their purchase behavior, which is expressed as

$$\tilde{c}_{kj}=\tilde{c}_{kj}(q_{kj}),\forall k,j \tag{30}$$

At this time, the objective function of the consumer is

$$Max\ Z_j=\sum_{j=1}^{J}u_{kj}(p_{kj},q_{kj})-\sum_{k=1}^{K}p_{kj}q_{kj}-\sum_{k=1}^{K}\tilde{c}_{kj}(q_{kj}) \tag{31}$$

$$\sum_{k=1}^{K}q_{kj}=d_j(p) \tag{32}$$

$$\forall k,j\ \ ,q_{kj}\geqslant 0.$$

Assuming that the above functions are all continuous and convex, according to the theory of variational inequality, (7) can be reduced to the following variational inequality

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\frac{\partial\tilde{c}_{kj}(q^*_{kj})}{\partial q_{kj}}+p_{kj}-\frac{\partial u_{kj}(p^*_{kj},q^*_{kj})}{\partial q_{kj}}-\zeta^*_{3j}\right)\times(q_{kj}-q^*_{kj})+\\&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(q^*_{kj}+\zeta^*_{3j}-\frac{\partial u_{kj}(p^*_{kj},q^*_{kj})}{\partial p_{kj}}\right)\times(p_{kj}-p^*_{kj})+\\&\sum_{j=1}^{J}\left(\sum_{k=1}^{K}q^*_{kj}-d_j(p^*_{kj})\right)\times(\zeta_{3j}-\zeta_{^*3j})\geqslant 0\end{aligned} \tag{33}$$

ζ_{3j} is the Lagrangian coefficient related to the constraint (23), and all ζ_{3j} forms the vector matrix ζ_3.

2.3. Model Dynamic Equilibrium Analysis Page

According to the related theorems of variational inequality, the model has a unique solution and satisfies the following conditions:

$$
\begin{aligned}
&\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial f_m(Q_1^*)}{\partial q_{mn}}+\frac{\partial c_{1mn}(q_{mn}^*,h_{mn}^*)}{\partial q_{mn}}-p_{mn}^*+\alpha_m\frac{\partial r_{mn}(q_{mn}^*,h_{mn}^*)}{\partial q_{mn}}\right)\times(q_{mn}-q_{mn}^*)+\\
&\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial c_{1mn}(q_{mn}^*,h_{mn}^*)}{\partial h_{mn}}-\frac{\partial v_{1mn}(h_{mn}^*)}{\partial h_{mn}}+\alpha_m\frac{\partial r_{mn}(q_{mn}^*,h_{mn}^*)}{\partial h_{mn}}\right)\times(h_{mn}-h_{mn}^*)+\\
&\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial c_{2mn}(q_{mn}^*,h_{mn}^*)}{\partial q_{mn}}+\frac{\partial \alpha r_{2mn}(q_{mn}^*,h_{mn}^*)}{\partial q_{mn}}+p_{mn}^*-\zeta_{1n}\right)\times(q_{mn}-q_{mn}^*)+\\
&\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial c_{2mn}(q_{mn}^*,h_{mn}^*)}{\partial h_{mn}}+\frac{\partial \alpha r_{2mn}(q_{mn}^*,h_{mn}^*)}{\partial h_{mn}}+\frac{\partial v_{2mn}(h_{mn}^*)}{\partial h_{mn}}\right)\times(h_{mn}-h_{mn}^*)+\\
&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{2nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}+\frac{\partial r_{2nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}-p_{nk}^*+\zeta_{1n}^*\right)\times(q_{nk}-q_{nk}^*)+\\
&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{2nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}+\frac{\partial r_{2nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}-\frac{\partial v_{2nk}(h_{nk})}{\partial h_{nk}}\right)\times(h_{nk}-h_{nk}^*)+\\
&\sum_{n=1}^{N}\left(\sum_{m=1}^{M}q_{mn}^*-\sum_{k=1}^{K}q_{nk}^*\right)\times(\zeta_{1n}-\zeta_{1n}^*)+\sum_{k=1}^{K}\left(\sum_{n=1}^{N}q_{nk}^*-\sum_{j=1}^{J}q_{kj}^*\right)\times(\zeta_{2k}-\zeta_{2k}^*)+\\
&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}+\alpha_k\frac{\partial r_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial q_{nk}}+p_{nk}^*-\zeta_{2k}^*\right)\times(q_{nk}-q_{nk}^*)+\\
&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}+\alpha_k\frac{\partial r_{3nk}(q_{nk}^*,h_{nk}^*)}{\partial h_{nk}}+\frac{\partial v_{3nk}(k_{nk}^*)}{\partial h_{nk}}\right)\times(h_{nk}-h_{nk}^*)+\\
&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\frac{\partial c_{kj}(q_{kj}^*)}{\partial q_{kj}}-p_{kj}^*+\zeta_{2k}^*\right)\times(q_{kj}-q_{kj}^*)+\sum_{j=1}^{J}\left(\sum_{k=1}^{K}q_{kj}^*-d_j(p_{kj}^*)\right)\times(\zeta_{3j}-\zeta_{*3j})+\\
&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\frac{\partial \tilde{c}_{kj}(q_{kj}^*)}{\partial q_{kj}}+p_{kj}-\frac{\partial u_{kj}(p_{kj}^*,q_{kj}^*)}{\partial q_{kj}}-\zeta_{3j}^*\right)\times(q_{kj}-q_{kj}^*)+\\
&\sum_{k=1}^{K}\sum_{j=1}^{J}\left(q_{kj}^*+\zeta_{3j}^*-\frac{\partial u_{kj}(p_{kj}^*,q_{kj}^*)}{\partial p_{kj}}\right)\times(p_{kj}-p_{kj}^*)\geqslant 0
\end{aligned}
\tag{34}
$$

Through the gradient projection algorithm, the variational inequality solution with continuous convex function can be solved:

$$
\begin{aligned}
\overline{q}_{mn}^{T-1}=\max\Bigg(0,\overline{q}_{mn}^{T-1}-\theta\Bigg(&\frac{\partial f_m(Q_1^{T-1})}{\partial q_{mn}}+\frac{\partial c_{1mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial q_{mn}}+\alpha_m\frac{\partial r_{1mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial q_{mn}}\\
&\frac{\partial c_{2mn}(q_{mn}^{T-1},h^{T-1})}{\partial q_{mn}}+\alpha_n\frac{\partial r_{2mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial q_{mn}}-\zeta_{1n}^{T-1}\Bigg)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
\overline{q}_{nk}^{T-1}=\max\Bigg(0,\overline{q}_{nk}^{T-1}-\theta\Bigg(&\frac{\partial c_{2nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial q_{mn}}+\alpha_n\frac{\partial r_{2nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial q_{mn}}-\zeta_{2k}^{T-1}\\
&\frac{\partial c_{3nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial q_{nk}}+\alpha_k\frac{\partial r_{3nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial q_{nk}}\Bigg)\Bigg)
\end{aligned}
$$

$$\overline{q}_{kj}^{T}=\max\left(0,\overline{q}_{kj}^{T-1}-\theta\left(\frac{\partial f_k(Q_3{}^{T-1})}{\partial q_{kj}}+\frac{\partial c_{kj}(q_{kj}^{T-1})}{\partial q_{kj}}+\frac{\partial\tilde{c}_{kj}(q_{kj}^{T-1})}{\partial q_{kj}}-\zeta_{3j}^{T-1}\right.\right.$$

$$\left.\left.-\frac{\partial u_{kj}(p_{kj}^{T-1},q_{kj}^{T-1})}{\partial q_{kj}}\right)\right)$$

$$\overline{p}_{kj}^{T}=\max\left(0,p_{kj}^{T-1}-\theta\left(q_{kj}^{T-1}-\frac{\partial u_{kj}(p_{kj}^{T-1},q_{kj}^{T-1})}{\partial p_{kj}}+\zeta_{3j}^{T-1}\right)\right)$$

$$\overline{h}_{mn}^{T}=\min\left(1,\max\left(0,h_{mn}^{T-1}-\theta\left(\frac{\partial c_{1mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial h_{mn}}+\frac{\partial v_{1mn}(h_{mn}^{T-1})}{\partial h_{mn}}+\right.\right.\right.$$

$$\alpha_m\frac{\partial r_{1mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial h_{mn}}+\frac{\partial c_{2mn}(q_{mn}^{T-1},h^{T-1})}{\partial h_{mn}}+\alpha_n\frac{\partial r_{2mn}(q_{mn}^{T-1},h_{mn}^{T-1})}{\partial h_{mn}}$$

$$\left.\left.\left.+\frac{\partial v_{2mn}(h_{mn}^{T-1})}{\partial h_{mn}}\right)\right)\right)$$

$$\overline{h}_{nk}^{T}=\min\left(1,\max\left(0,h_{nk}^{T-1}-\theta\left(\frac{\partial c_{1nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial h_{nk}}+\frac{\partial v_{1nk}(h_{nk}^{T-1})}{\partial h_{nk}}+\right.\right.\right.$$

$$\alpha_n\frac{\partial r_{1nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial h_{nk}}+\frac{\partial c_{2nk}(q_{nk}^{T-1},h_{nk}{}^{T-1})}{\partial h_{nk}}+\alpha_k\frac{\partial r_{2nk}(q_{nk}^{T-1},h_{nk}^{T-1})}{\partial h_{nk}}$$

$$\left.\left.\left.+\frac{\partial v_{2nk}(h_{nk}^{T-1})}{\partial h_{nk}}\right)\right)\right)$$

$$\overline{\zeta}_{1n}^{T}=\max\left(0,\zeta_{1n}^{T-1}-\theta\left(\sum_{m=1}^{M}q_{mn}^{T-1}-\sum_{k=1}^{K}q_{nk}^{T-1}\right)\right)$$

$$\overline{\zeta}_{2k}^{T}=\max\left(0,\zeta_{2k}^{T-1}-\theta\left(\sum_{n=1}^{N}q_{nk}^{T-1}-\sum_{j=1}^{J}q_{kj}^{T-1}\right)\right)$$

$$\overline{\zeta}_{3j}^{T}=\max\left(0,\zeta_{3j}^{T-1}-\theta\left(\sum_{k=1}^{K}q_{kj}^{T-1}-d_j(p_{kj}^{T-1})\right)\right)$$

2.4. Numerical Examples

A network of two raw material suppliers, two manufacturers, two retailers, and zero consumer markets.

To simplify the calculation, assume that the transaction cost is 0 and the weight is 1. Other functions of positioning in the model are set as follows:

The procurement cost of the raw material supplier is:

$$f_1(Q_1)=\left(\sum_{n=1}^{2}q_{1n}\right)^2+0.5\left(\sum_{n=1}^{2}q_{2n}\right)\left(\sum_{n=1}^{2}q_{1n}\right)+2\left(\sum_{n=1}^{2}q_{1n}\right)$$

$$f_2(Q_1)=0.5\left(\sum_{n=1}^{2}q_{2n}\right)^2+\left(\sum_{n=1}^{2}q_{1n}\right)\left(\sum_{n=1}^{2}q_{1n}\right)+\left(\sum_{n=1}^{2}q_{2n}\right)$$

The manufacturer's production costs are:

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$$f_1(Q_3) = \left(\sum_{j=1}^{2} q_{1j}\right)^2 + 2\left(\sum_{j=1}^{2} q_{1j}\right)\left(\sum_{j=1}^{2} q_{2j}\right) + 4$$

$$f_2(Q_3) = 2\left(\sum_{j=1}^{2} q_{1j}\right)^2 + \left(\sum_{j=1}^{2} q_{1j}\right)\left(\sum_{j=1}^{2} q_{2j}\right) + 3$$

The economic cost function is

$$v_{1mn}(h_{mn}) = 1.5h_{mn} + 2,\ \ \forall m,n$$

$$v_{2mn}(h_{mn}) = 2h_{mn} + 1,\ \ \forall m,n$$

$$v_{2nk}(h_{nk}) = h_{nk} + 2, \forall n,k$$

$$v_{3nk}(h_{nk}) = h_{nk} + 2, \forall n,k$$

The risk function is:

$$r_{1mn}(q_{mn},h_{mn}) = {q_{mn}}^2 + q_{mn} - h_{mn}, \forall m,n$$

$$r_{2mn}(q_{mn},h_{mn}) = q_{mn}^2 + 2q_{mn} - h_{mn}, \forall m,n$$

$$r_{2nk}(q_{nk},h_{nk}) = 2q_{nk}^2 + q_{nk} - 1.5h_{nk}, \forall n,k$$

$$r_{3nk}(q_{nk},h_{nk}) = q_{nk}^2 + q_{nk} - 2h_{nk}, \forall n,k$$

The consumer utility function is:

$$u_{kj}(p_{kj},q_{kj}) = 0.5 = 0.5\,p_{kj}q_{kj}, \forall k,j$$

The market demand is:

$$d_1(p) = -\,p_{11} - 2p_{21} + 1000$$

$$d_2(p) = -\,p_{12} - 2p_{22} + 1000$$

The parameters are set to a: step length b, loop verification condition c. Using MATLAB programming, the final calculation results are as follows:

$Q_1: q_{111} = 17.2792 \quad q_{112} = 17.2792 \quad q_{121} = 17.2792 \quad q_{122} = 17.2792$

$q_{211} = 23.2056 \quad q_{212} = 23.2056 \quad q_{221} = 23.2056 \quad q_{222} = 23.2056$

$h_1: h_{111} = h_{112} = h_{121} = h_{122} = h_{211} = h_{212} = h_{221} = h_{222} = 0$

$Q_2: q_{111} = 24.1233 \quad q_{112} = 24.1233 \quad q_{121} = 16.3616 \quad q_{122} = 16.3616$

$q_{211} = 24.1233 \quad q_{212} = 24.1233 \quad q_{221} = 16.3616 \quad q_{222} = 16.3616$

$h_2: h_{111} = h_{112} = h_{121} = h_{122} = h_{211} = h_{212} = h_{221} = h_{222} = 1$

$Q_3: q_{11} = 0 \quad q_{12} = 96.4933 \quad q_{21} = 65.4622 \quad q_{22} = 0$

$p: p_{11} = 333.6047 \quad p_{12} = 333.6904 \quad p_{21} = 358.6047 \quad p_{22} = 333.8321$

$\zeta_1: \zeta_{11} = \zeta_{12} = 258.7620$

$\zeta_2: \zeta_{21} = 405.5019 \quad \zeta_{22} = 358.9313$

$\zeta_3: \zeta_{31} = \zeta_{32} = 0$

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From the above results, it can be seen that the amount of raw materials provided by raw material suppliers to manufacturers is not much different from the total amount of materials provided by manufacturers to consumers. The material flow of materials traded between each layer basically conforms to the set quantity relationship.

3. Conclusion

This paper studies the problem of supply chain's overall response to risks. Firstly, the equilibrium model of the supply chain network is established when considering the risk function, and the conditions of the equilibrium state of the supply chain network are obtained by using the variational inequality, and then the improved projection gradient algorithm is used to solve the model. And use numerical examples to verify the model,The article adds a risk function to the basic model to better study the impact of risk on the supply chain.

With the further development of the global economy and closer cooperation between enterprises and enterprises, the influence of external factors on the supply chain will also increase. In the actual production of enterprises, the cost of risk is not only reflected in the cost of deviation, but also involves factors such as the break of the capital chain, the withdrawal of manufacturers, and the serious shortage of raw materials, which need to be further discussed in future research.

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