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# A Fuzzy Sliding-mode PID Controller for Flight Trajectory of Quadrotors under Wind Disturbances

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## Abstract

**Based on the three-dimensional model of the UAV, this paper constructs a complete rigid body dynamics model of the quadrotor UAV, and on this basis controls the flight trajectory of the quadrotor. At the same time, this article takes the quay crane as the background to establish the wind interference model of the port, and then the dual Closed-loop system controls the position and attitude of the quad-rotor UAV, including the backstepping sliding control of the attitude loop and the traditional position loop pid control, in order to realize that the drone can better inspect the measuring points of the quay crane. The simulation results show that the UAV trajectory control designed by this algorithm has good tracking performance, and the UAV can quickly reach the established posture and accurately reach the planned position.**

## Keywords

**Double Closed Loop System; Back-stepping Sliding Control; Intelligent Proportion-Integral-Derivative (PID) Algorithm; Position and Attitude Control.**

## 1. Introduction

UAV is the abbreviation of Unmanned Aerial Vehicle (Unmanned Aerial Vehicle), also known as "UAV", which is a new force in modern aerial vehicles. In recent years, with the advancement of technology, the performance of UAVs has been greatly improved, and new types of UAVs with high speed, miniaturization and high endurance have emerged one after another, and the application range of UAVs has also been expanding. In the military field, UAVs can be used to train target drones, conduct military reconnaissance, induce interference, and drop bombs. In civilian use, UAVs can be used for aerial photography, surveying and mapping, environmental monitoring and other applications. Due to its low cost and easy maintenance, UAVs have attracted more and more attention.

For the control system of the quad-rotor UAV, the quad-rotor UAV control system is a complex under-driving system with specific characteristics such as strong coupling, multi-variable, nonlinear and so on. In addition, it suffers from various physical effects, such as gravity and gyro effects. It is also susceptible to interference from external environments such as air. Therefore, it is an inevitable requirement to control the flight attitude accurately and reliably. At present, many effective control methods, such as PID control, adjust the deviations obtained by integrating, differential and proportional coefficients, so that the system can respond quickly[1]; fuzzy PID control, fuzzy PID parameters are established by establishing fuzzy rules to achieve PID parameters However, the actual adjustment process is applied to complex systems, and the adjustment effect is not very good[2]; expert PID control and intelligent PID control[3] are also based on the expert library[4] and intelligent rules established by the deviation range to adjust the PID parameters, and the control effect is limited; The synovial variable structure control can realize the decoupling of the strong coupling system[5]; the PI adaptive control[6] and the L1 adaptive control[7] can independently adjust the corresponding parameters to achieve better control effects; the adaptive synovial control can perform the decoupling of the strong coupling system. Decoupling and realizing the automatic adjustment ability of control parameters [8].

The quad-rotor UAV is an under-driven strong coupling system. In the control field, the control research is mainly carried out for the strong coupling part, and the sliding mode control, PID control, etc. are mainly used for decoupling and control. Trajectory tracking mainly uses a combination of multiple algorithm control algorithms, such as Back-stepping control and L1 adaptive control. Aiming at the uncertainties such as external interference and internal parameter perturbation in the inner loop, the L1 adaptive control idea is introduced to compensate for its influence; Step control and adaptive sliding film control realize stable tracking control of position information under external disturbances and stable tracking control of attitude information under uncertain parameters and external disturbances. Fuzzy PID control combined with genetic algorithm processing realizes wind disturbances under the circumstances, and the problem of drone jitter is solved. The dynamic mask method based on the position loop and the high-order sliding mode integral based on the attitude loop apply the discontinuous control input to the high-order differential of the sliding mode surface, so that the actual control amount is Essentially continuous, to a large extent suppress or even eliminate the chattering, the Lyapunov function design method and the establishment of the visual simulation of the UAV are observed from time to time.

Based on the existing research results, this paper uses the PID algorithm and the inversion sliding mode control algorithm as the basis, introduces dynamic surface control, and designs the dual Closed-loop control system of the quad-rotor UAV. The designed controller includes the inversion sliding film control of the attitude and Fuzzy PID control of the position loop. In the position loop, this paper implements the PID parameter self-adjustment function of the position control through the fuzzy processing and de-fuzzification process of the feedback amount deviation, and ensures the reliability of the solution process from the position to the attitude; in the attitude loop, through inversion Sliding mode control decouples the system, decomposes the complex nonlinear system of the quadrotor UAV into three linear subsystems, and realizes the effective control of the attitude loop. The proposed control strategy can effectively realize the asymptotic tracking and interference suppression of the trajectory, and realize the global finite time convergence. The main work of this paper includes: 1) Using the output regulator based on fuzzy PID algorithm to realize the position calculation of the position subsystem; 2) the method controls the attitude accurately, weakens the system chattering by using the Back-stepping sliding mode control, and achieves finite time convergence.

## 2. The Dynamic Model and Parameters of UAV

### 2.1. Dynamic Model of UAV

Figure 1 below is a schematic diagram of a cruciform quadrotor UAV:

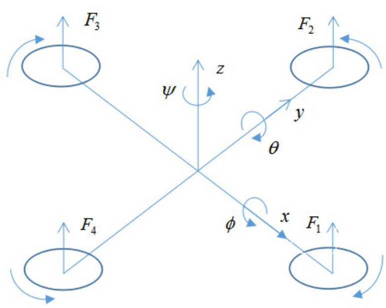


**Fig 1.** Schematic diagram of quadrotor UAV model

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The establishment of the quad-rotor UAV model mainly relies on the conversion relationship between the ground coordinate system and the body coordinate system. The following assumptions are made for the quad-rotor UAV:

(1) The four-rotor UAV is a completely symmetrical rigid body, ignoring its structure and elastic deformation;

(2) The body coordinate system coincides with the center of mass of the quad-rotor UAV;

(3) Assume that the ground coordinate system is an inertial coordinate system.

## 2.2. Lift Coefficient and Torque Coefficient of UAV

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The formula of propeller pulling force:

$$T_p = (\frac{1}{2\pi})^2 C_T \rho \varpi^2 (2r_p)^4 \tag{1}$$

The formula of torque:

$$M_p = (\frac{1}{2\pi})^2 C_M \rho \varpi^2 (2r_p)^5 \tag{2}$$

among them $\rho$ is the air density, $\varpi$ is the rotational angular velocity of the propeller, and $C_T, C_M$ is the tensile force coefficient and the torque coefficient respectively.

The formula of $\rho$ :

$$\rho = \frac{273P_a}{101325(273+T_t)}\rho_0 \tag{3}$$

where $\rho_0 = 1.293kg/m^3$ is the standard atmospheric density.

The formula of atmospheric pressure:

$$P_a = 101325(1-0.0065\frac{h}{273+T_t})^{5.5261} \tag{4}$$

In the case of low flight altitude, atmospheric temperature and altitude values can be input as a constant into the estimation model.

And the estimation formula of $C_T, C_M$ can be expressed as:

$$\begin{cases} C_T = f_{C_T}(\Theta_p) \\ C_M = f_{C_M}(\Theta_p) \end{cases} \tag{5}$$

Then

$$C_T = 0.25\pi^3 \lambda \varsigma^2 B_p K_0 \frac{\varepsilon \arctan \frac{H_p}{\pi D_p} - \alpha_0}{\pi A + K_0}$$
$$C_M = \frac{1}{4A}\pi^2 C_d \varsigma^2 \lambda^2 B_p \tag{6}$$

Where

$$C_d = C_{fd} + \frac{\pi A K_0^2}{e} \frac{(\varepsilon \arctan(H_p / \pi D_p) - \alpha_0)^2}{(\pi A + K_0)^2} \tag{7}$$

The range of values for these parameters is as follows.

$$\begin{cases} A = 5 \sim 8, \varepsilon = 0.85 \sim 0.95, \lambda = 0.7 \sim 0.9 \\ \varsigma = 0.4 \sim 0.7, e = 0.7 \sim 0.9, C_{fd} = 0.015 \\ \alpha_0 = -\pi / 36 \sim 0, K_0 = 6.11 \end{cases} \tag{8}$$

The following table 1 below are the corresponding parameters of the UAV simulated in this article.

So, the conversion matrix R from the body coordinate system to the inertial coordinate system is as shown:

$$R = \begin{bmatrix} \cos\theta\cos\psi & \sin\theta\cos\psi\sin\phi - \sin\psi\cos\phi & \sin\theta\cos\psi\cos\phi + \sin\psi\sin\phi \\ \cos\theta\sin\psi & \sin\theta\sin\psi\sin\phi + \cos\psi\cos\phi & \sin\theta\sin\psi\cos\phi - \cos\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

**Table 1.** The Parameter of the UAV

| name | Symbol | Value |
|---|---|---|
| Acceleration of gravity | g | 9.8100(m/s^2) |
| Wheelbase | l | 0.2223(m) |
| Mass of the UAV | m | 1.0230(kg) |
| Force coefficient | CT | 1.4865e-07(N/RPM^2) |
| Torque coefficient | CM | 2.9250e-09(N*m/RPM^2) |
| Moment of inertia in x | $J_x$ | 0.0095(kg/m^2) |
| Moment of inertia in y | $J_y$ | 0.0095(kg*m^2) |
| Moment of inertia in z | $J_z$ | 0.0186(kg*m^2) |

According to the torque of the four motors of the drone, the four inputs are controlled: U1, U2, U3, and U4, the four inputs control the position (x, y, z) and the angle of the drone. The six outputs. Among them, x, y, and z represent the coordinates on the x-axis, y-axis, and z-axis of the drone, respectively, and φ, θ, and ψ represent the roll angle, pitch angle, and yaw angle of the drone, respectively. The input control quantity is expressed as:

$$\begin{cases} U_1 = C_T(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ U_2 = C_T(\omega_4^2 - \omega_2^2) \\ U_3 = C_T(\omega_3^2 - \omega_1^2) \\ U_4 = C_M(\omega_4^2 - \omega_3^2 + \omega_2^2 - \omega_1^2) \end{cases} \tag{9}$$

where $\omega_i$ (i=1,2,3,4) represents the speed of motor.

Build the UAV dynamics model:

$$\begin{cases} \ddot{x} = U_1(\cos\phi\sin\theta\sin\psi + \sin\phi\sin\psi)/m \\ \ddot{y} = U_1(\cos\phi\sin\theta\sin\psi - \sin\phi\sin\psi)/m \\ \ddot{z} = U_1(\cos\phi\cos\theta)/m - g \\ \ddot{\phi} = \dot{\theta}\dot{\psi}(I_y - I_z)/I_x + lU_2/I_x \\ \ddot{\theta} = \dot{\phi}\dot{\psi}(I_z - I_x)/I_y + lU_3/I_y \\ \ddot{\psi} = \dot{\theta}\dot{\psi}(I_x - I_y)/I_z + lU_4/I_z \end{cases} \tag{10}$$

# 3. The Design of the Controller

## 3.1. The Design of the Attitude Controller

The attitude control of the quad-rotor UAV is the most important content in the entire UAV control system. The purpose of the design of the attitude control loop is to allow the flight attitude of the UAV to track the desired attitude calculated by the trajectory control loop, that is, the quadrotor UAV can achieve consistent flight attitude with the desired attitude under various flight conditions.

The attitude loop is the inner loop of the trajectory loop. The output of the trajectory control loop is the input of the attitude loop, that is, the desired attitude angle, and the actual attitude angle of the UAV is used as the feedback value. The inner loop output corresponds to the 4 inputs of the UAV the variable is the speed of the 4 motors.

We define the state output vector of the UAV as:

$$X = (x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \varphi, \dot{\varphi}) \tag{11}$$

The system state vector is:

$$Y = (x, y, z, \phi, \theta, \varphi) \tag{12}$$

Letting

$$\begin{aligned} &x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}\, x_5 = z, x_6 = \dot{z}, \\ &x_7 = \phi, x_8 = \dot{\phi}\, x_9 = \theta, x_{10} = \dot{\theta}, x_{11} = \varphi, x_{12} = \dot{\varphi}, \end{aligned} \tag{13}$$

Thus, the state space equation can be expressed as

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$$Y = CX \tag{14}$$

Then the control system of the roll angle can be expressed as:

$$\begin{cases} \dot{x}_7 = x_8 \\ \dot{x}_8 = f(x_{10}, x_{12}) + b_1 U_2 \\ y_7 = x_7 \end{cases} \tag{15}$$

In the process of designing the control rate, we need to introduce the required roll angle $x_7$ and the roll angle at this moment $x_{7d}$. Currently, the deviation of the roll angle $z_7 = x_7 - x_{7d}$.
So, the Lyapunov function is constructed as follows:

$$V_1 = \frac{1}{2} z_7^2 \tag{16}$$

Deriving it to get:

$$\dot{V}_1 = z_7 \dot{z}_7 = z_7 (x_8 - \dot{z}_{7d}) \tag{17}$$

The approach rate of the synovial surface of the roll angle is:

$$s_7 = k_7 z_7 + z_8 \tag{18}$$

Then the Lyapunov function is built as follows:

$$V_2 = V_1 + \frac{1}{2} s_7^2 \tag{19}$$

So, we can design the Back-stepping sliding mode control law of the roll angle:

$$U_2 = -\frac{1}{b_1}[\frac{1}{k_7} z_8 + k_7 (z_8 - c_7 z_7) + f(x_{10}, x_{12}) - u_7 + h_7 s_7] \tag{20}$$

where

$$\dot{V}_2 \le -c_7 z_7^2 - \frac{1}{k_7} z_8^2 - h_7 s_7^2 \le 0 \tag{21}$$

Therefore, the system designed according to this control rate is gradually stable. In the same way, the control rate of pitch angle and yaw angle can be designed as shown below:

$$U_3 = -\frac{1}{b_2}[\frac{1}{k_9} z_{10} + (c_9 + k_9)(z_{10} - x_{9d}) + x_8 x_{12} a_3 + x_8 a_4 - \ddot{x}_{9d} + h_9 s_9] \tag{22}$$

ISSN: 2688-9323

$$U_4 = -\frac{1}{b_2}[\frac{1}{k_{11}} z_{12} + (c_{11} + k_1)(z_{12} - x_{11d}) + x_{10} x_{12} a_5 - \ddot{x}_{11d} + h_{11} s_{11}] \tag{23}$$

## 3.2. The Design of the Position Controller

The purpose of designing the trajectory control loop is to allow the UAV to track a given position trajectory quickly, stably and accurately. The trajectory control loop takes the expected flight trajectory as input, the actual position of the quadrotor UAV as feedback, and the three attitude angles of the quadrotor UAV as output. The trajectory control loop needs to calculate the attitude angle signal required by the attitude control loop, and then the attitude control loop completes the tracking control of the attitude angle.

First, use the proportional control of the difference between the desired position and the actual position to calculate the linear velocity of the UAV along the X and Y directions, and then use the proportional integral control of the difference between the desired linear velocity and the actual linear velocity to calculate the desired attitude angle. The input of the trajectory control loop is Xd, Yd, the output is the expected value of the attitude angle in the X and Y directions, and the following is the speed in the X, Y direction converted into the inertial coordinate system:

$$\begin{cases} V_{xd} = (X_d - X) * \cos(\phi) + (Y_d - Y) * \sin(\phi) \\ V_{yd} = (Y_d - Y) * \cos(\phi) - (X_d - X) * \sin(\phi) \end{cases} \tag{24}$$

So, the design of the position loop control law is as follows:

$$\begin{cases} \theta_d = K_{px}(V_{xd} - V_x) + K_{ix} \int (V_{xd} - V_x) dt + K_{dx}(V_{xd} - V_x) \\ \varphi_d = K_{py}(V_{yd} - V_y) + K_{iy} \int (V_{yd} - V_y) dt + K_{dy}(V_{yd} - V_y) \end{cases} \tag{25}$$

# 4. Simulation

The Simulink model of the double Closed-loop control system is shown in Figure 2 below.

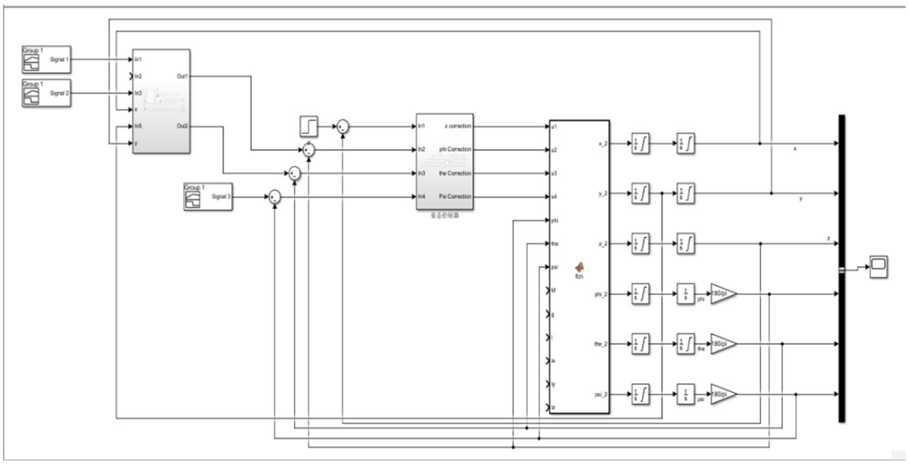

**Fig 2.** The construction of simulation

## 4.1. Comparison of Closed-loop Response of Position Loop Single System

The response curve of the control system for its z-direction displacement is shown in Figure 3 below without any control algorithm.

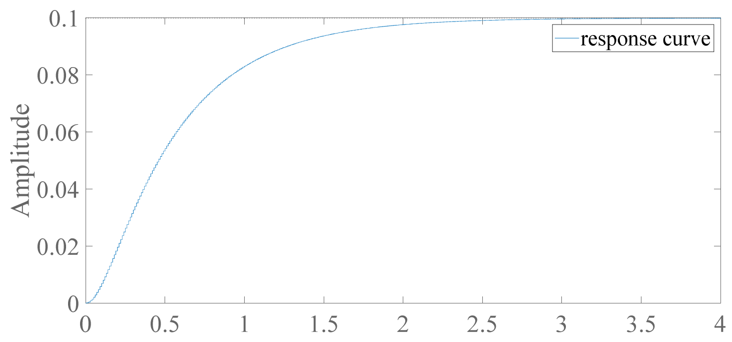


**Fig 3.** Response curve of Closed-loop system without control algorithm

It can be seen from the system response curve that the system response is slow, which does not meet the requirements of the control system.

Perform fuzzy pid control on the position control subsystem, and compare the difference between traditional pid control and fuzzy pid control under the same Kp, Ki, and Kd values. Run the fuzzing program through MATLAB to get the curve in Figure 4 as follows:

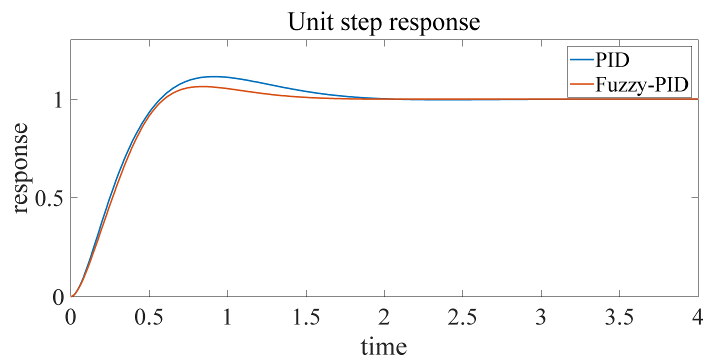


**Fig 4.** Comparison curve of PID algorithm and fuzzy PID algorithm in the horizontal direction

It can be seen from the figure that after using fuzzy pid control, the response curve of the system is obviously better than before, and the overshoot of the system is greatly reduced. The response is almost unchanged, which enables the system to become more stable.

## 4.2. The Response Curve of the Individual Synovial Control of the Attitude Loop

Figure 5 below is the control input for the yaw angle and the corresponding curve of the system:

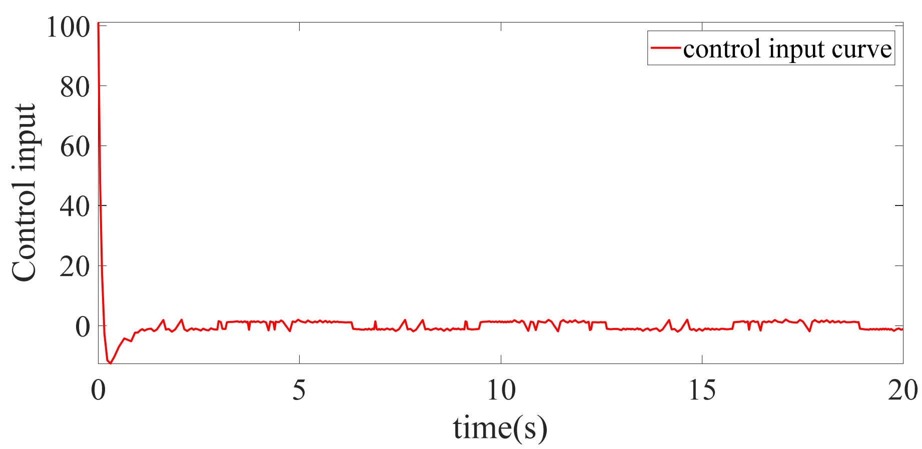


**Fig 5.** Yaw angle control input and output response curve

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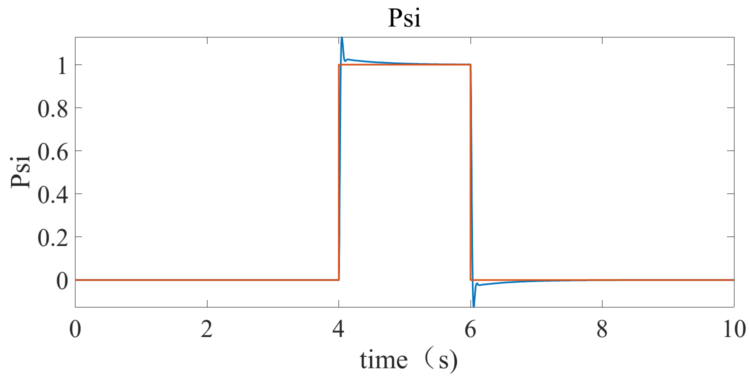
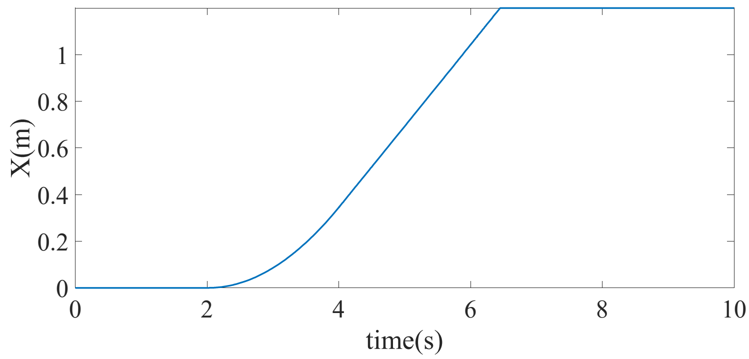
It can be seen from the figure that the control input obtained by the model in this paper has good input characteristics, and the output response speed is very fast, there is almost no overshoot, which can meet the control requirements we need.

## 4.3. Attitude Control Effect of the Entire System

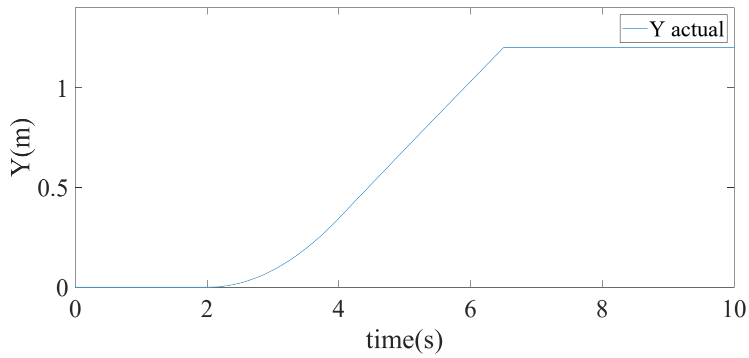
Figures 6 and 7 below are the adjustment effect diagrams of the designed attitude controller. The control goal is to require the UAV to rise to 4m in the z direction. The control requirements for the roll angle, pitch angle and yaw angle are to change $\phi$ to 1° at t=4s, and at t=6s.Change to 0, change $\theta$ to 1° at t=2s, change to 0 at t=4s, change $\psi$ to 1° when t=4s, and change to 0 when t=6s.




(a)




(b)

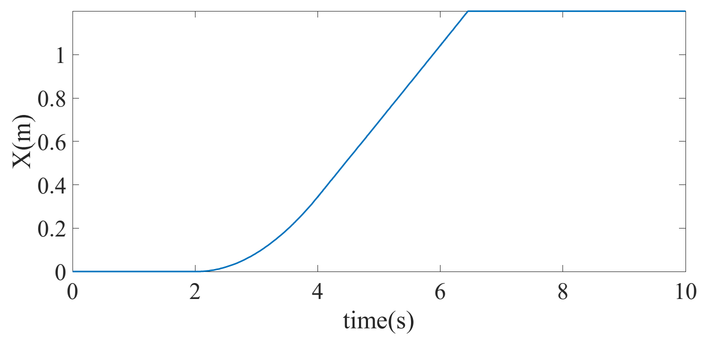



(c)

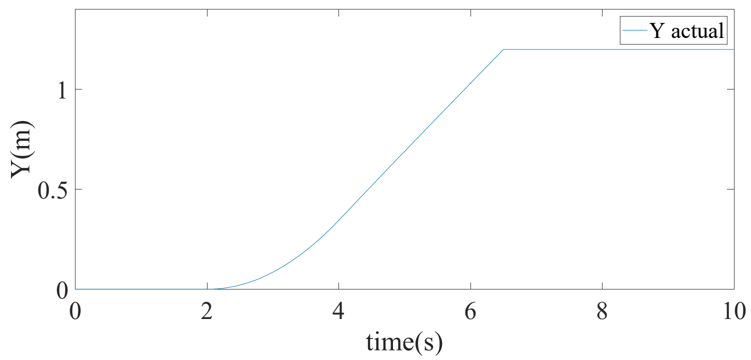
**Fig 6.** Tracing curve of three Euler angle

According to Figure 6, it can be seen that the attitude control has achieved the expected effect very well, the result meeting our control requirements.

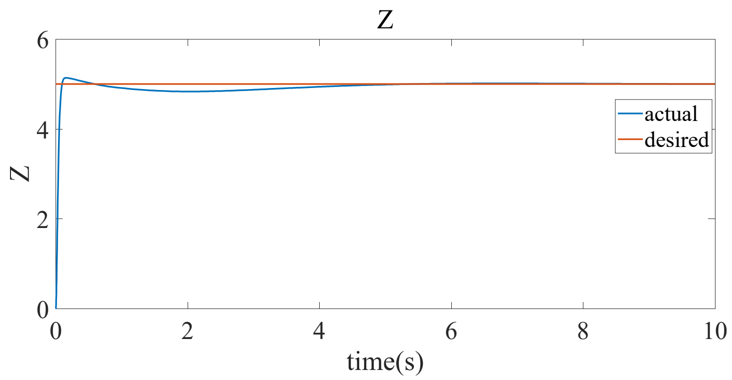
Figure 7 below is the position change curve under attitude control. It can be seen that the z direction has reached our expected requirements in a very short time, and the x, y direction has a small change in attitude angle, so the displacement in the x, y direction changes slowly until t= At 6s, the attitude angle is all 0. At this time, the position of the drone is stationary and hovering.



(a)



(b)



(c)

**Fig 7.** Position control input and output response curve

## 5. Summary

The results show that although the traditional PID control can also achieve basic control of the quadrotor UAV, it takes a long time to adjust the attitude when encountering higher frequency and larger interference, and the controller cannot adjust the quadcopter in time. The attitude of the rotary-wing UAV restores it to the desired state, and there will be a large range of vibrations during the adjustment process, and even out of control, which is not enough to meet our requirements for the quad-rotor UAV. The controller optimized by the dual Closed-loop control algorithm has stronger anti-interference ability, and can quickly and stably maintain

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the attitude of the quad-rotor UAV in the presence of complex interference, and has high control accuracy. This proves that the controller designed in this paper has good control performance and robustness, and effectively improves the performance of the quadrotor UAV control system.

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