

# Green Marketing Strategies of Supply Chain and Incentive Contracts Considering Product Green Degree and Sales Effort

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## Abstract

This paper investigates a two-echelon supply chain consisting of a single manufacturer and a single retailer. Considering product green degree and sales effort, we study whether manufacturer and retailer take green marketing strategies and further examine how manufacturer and retailer establish incentive contract respectively under the equilibrium market to maximize profits. We construct Stackelberg game, and explore the impact of important parameters on the optimal decisions, and compare the optimal decisions in the incentive contract and in the non-incentive contract through numerical study. The results show that it is more advantageous for both manufacturer and retailer to choose to take green marketing strategy compared with only one member taking green marketing strategy. Besides, Cost-sharing contracts can effectively achieve supply chain coordination. When manufacturer and retailer choose the optimal cost sharing ratio, it is more beneficial for them to incentivize other partner to improve product green degree and increase sales effort, and thus realize profit maximization.

## Keywords

Green Marketing; Product Green Degree; Sales Effort; Incentive Contracts; Stackelberg Game.

## 1. Introduction

The prevalence of the concept of green consumption and the improvement of consumers' green preference have increased market demand for green products, thus promoting the development of green marketing in the supply chain. Green marketing usually includes green investment in products and green sales effort, for example, SAIC-GM takes green and smart manufacturing strategy, and Philips adopts Energy Star label for green campaign. In order to gain advantages in the fierce competition, the supply chain needs close cooperation of members to obtain higher market share. But the fact is that in the process of supply chain green marketing, when one of the members takes green marketing strategy, the others can obtain green marketing benefits for free. Therefore, it is of great significance to study whether members in the supply chain take green marketing strategies and how to coordinate the supply chain.

The research that considers green marketing in supply chain has achieved lots of results, which has been studied from two aspects of green investment in products and green sales effort. Some literature study the impact of green investment on supply chain. The green investment of manufacturer is mainly that use fewer resources to produce the same products and improve resource utilization (Liu 2017), and improve product green degree. Zhang and Liu (2013) studied the decision situation of members in three-echelon supply chain when market demand is related to product green degree. Basiri and Heydari (2017) examined the impact of product green degree and consumer environmental awareness on supply chain coordination. Ghosh and Shah (2012) considered a two-echelon supply chain consisting of a manufacturer and a retailer, where market demand is jointly determined by price and green degree, and coordinated using a two-part contract.

There are lots of literature examining the impact of sales effort on supply chain. The sales effort of retailer includes advertising investment, product display, sales staff explanation and demonstration to increase the market demand. Gao et al. (2016) considered sales efforts and explored the pricing decision problem of a closed-loop supply chain. Ma et al. (2017) studied the pricing decisions of closed-loop supply chain when retailer was responsible for sales effort, and discussed the impact of retailer's fairness concerns on sales effort. Zerang et al. (2018) examined that retailer was responsible for sales effort, and found that closed-loop supply chain led by manufacturer are the most efficient. Saha et al. (2019) considered the influence of price and sales effort level on product market demand and studied the channel coordination problem of three-echelon supply chain. Taleizadeh et al. (2016) studied the problem of closed-loop supply chain pricing decisions when both manufacturer and retailer make sales efforts.

The research on coordination of supply chain that the existing literature provide lots of contracts for guiding cooperation among supply chain partners. Hong and Guo (2019) found that when manufacturer shares green marketing costs with retailer, it may harm the benefits of retailer, but can promote green manufacturing. Ma et al. (2013) proposed a new contract combining two-part pricing contract and Cost-sharing contract to realize the coordination of supply chain. Taylor (2002) studied that when sales effort affects the demand, the combination of sales rebate contract and return contract can make retailer invest optimal promotion effort and achieve channel coordination. Swami and Shah (2013) found a two-part tariff contract can solve channel coordination problem on the efforts of a manufacturer and a retailer to promote green products. Krishnan et al. (2004) found that only combining buy-back contract with Cost-sharing agreements is the best to achieve channel coordination. Later, Tsao and Sheen (2012) examined promotion cost sharing is a critical mechanism to coordinate the supply chain.

Our research refers to Li et al. (2021), which established an evolutionary game model of mobile phone manufacturer and retailer's green marketing strategies involving BDTA, and discussed the influence of consumers' green preferences and BDTA on mobile phone manufacturer and retailer's green marketing strategies. Our paper differs from the mentioned literature in that firstly, most of the existing literature directly examine manufacturer and retailer will take green marketing strategy and discuss the impact of green efforts on the decision of members in supply chain, while we consider whether a general manufacturer and a general retailer take green marketing strategy and use Stackelberg game to solve the equilibrium solution in different market environment, making the research more general and comprehensive, and providing theoretical guidance for manufacturer and retailer to take optimal decisions. Then, we further examine the optimal incentive contract of manufacturer and retailer under the market equilibrium, and provide references for manufacturer and retailer to maximize profits by using incentive contract and coordinate the supply chain.

The rest of the paper is organized as follows. We present the problem description and hypothesis in Section 2. We characterize and analyze the case that when manufacturer or retailer takes green marketing in Section 3. We examine and analyze the incentive contracts of manufacturer and retailer under the equilibrium market in Section 4. We explore the numerical analysis in Section 5. We conclude the study and offer directions for future research in Section 6. To improve readability, all proofs are presented in Appendix.

## 2. Problem Description

This paper considers a two-echelon supply chain consisting of a single manufacturer and a single retailer, in which the manufacturer produces green products and wholesales green products to the retailer at price  $w$ , and then the retailer sells them to consumer at price  $p$ . We think manufacturer is the leader in supply chain and retailer is the follower, and the information between them is completely symmetrical. Both the manufacturer and the retailer can take

green marketing strategy. In the supply chain, when either manufacturer or retailer adopts green marketing strategy, the other can get green marketing benefits for free. Under consumers' green preference, if neither of them adopts green marketing strategy, the market share will decrease, which is obviously unprofitable for manufacturer and retailer. Therefore, this paper only discusses situations where at least one of them adopts green marketing. We assume the manufacturer and the retailer can decide to take green marketing strategy (T) or non-green marketing strategy (N), and both are bounded rationality. When manufacturer takes green marketing strategy, it means that the manufacturer will make green investment in products such as technological innovation, process adjustment, so we think manufacturer's green investment level is  $g$ , i.e., product green degree. The green investment cost is a concave function of the product green degree, assuming that is  $\frac{1}{2}\eta g^2$ , where  $\eta$  is product green cost coefficient. When retailer takes green marketing strategy, it means that retailer will make sales effort such as advertising, promotional activities, so we think the retailer's green input level is  $s$ , i.e., sales effort. The sales effort cost is a concave function of the sales effort level, assuming that is  $\frac{1}{2}\theta s^2$ , where  $\theta$  is sales effort cost coefficient.

When both manufacturer and retailer take green marketing strategy, the market demand depends on selling price, product green degree and sales effort. Higher product green degree and sales effort leads to more market demand, so we assume the market demand is  $D = a - bp + kg + ls$ , where  $a$  is the potential market demand,  $b$  is consumers' sensitivity coefficient to selling price,  $k$  is the product green effect, indicating the impact of product green degree on demand,  $l$  is the sales effort effect, indicating the impact of sales effort on demand. When  $a = bp$ , it means that product demand is only related to consumers' green preferences. In order to ensure the market demand is in line with the actual situation, we assume  $a > bp$ . In addition, we assume the manufacturer's unit production cost is  $c$ . We think consumers are all rational, their sensitivity to selling price should be within a reasonable range, assuming  $\frac{l^2\eta + k^2\theta}{\eta\theta} < b < \frac{a}{c}$ .

According to the above problem description and assumptions, this paper will discuss whether manufacturer and retailer take green marketing strategy, so we consider the following models:

**Table 1.** Summary of Notation

Parameters	Definition
$a$	Potential market demand
$b$	Consumers' sensitivity coefficient to selling price
$k$	Product green effect
$l$	Sales effort effect
$\eta$	Product green cost coefficient
$\theta$	Sales effort cost coefficient
$c$	Manufacturer's unit production cost
$\lambda$	Manufacturer's share of retailer's sales effort cost
$D^j$	Market demand in case $j$
$\pi_M^j$	Profit of manufacturer in case $j, j \in \{TN, NT, TT, TZ, TL\}$
$\pi_R^j$	Profit of retailer in case $j$
Superscript *	Optimal decisions for decision variables
Decision variable	
$w^j$	Manufacturer's wholesale price in case $j$
$p^j$	Retailer's selling price in case $j$
$g^j$	Manufacturer's product green degree in case $j$
$s^j$	Retailer's sales effort in case $j$

(1) manufacturer takes green marketing strategy but retailer does not (Case TN); (2) retailer takes green marketing strategy but manufacturer does not (Case NT); (3) both manufacturer and retailer take green marketing strategy (Case TT).

The parameters used in the model are shown in the following table, see [Table 1](#).

### 3. The Model

In this section, we build and solve models in different scenarios described above, and analyze the equilibrium solutions of manufacturer and retailer in different cases.

#### 3.1. Only Manufacturer Takes Green Marketing (Case TN)

When manufacturer makes green investment in products but retailer does not make sales effort, the market demand will be reduced but the retailer can still benefit from the manufacturer's green marketing strategy and it does not need to pay sales effort cost.

Considering a two-stage Stackelberg game, firstly, the manufacturer decides simultaneously wholesale price  $w^{TN}$  and product green degree  $g^{TN}$ ; secondly, the retailer decides selling price  $p^{TN}$  based on the manufacturer's optimal decision. The market demand is

$$D^{TN} = a - bp^{TN} + kg^{TN} \quad (1)$$

The profit function of manufacturer and retailer respectively are

$$\pi_M^{TN}(w^{TN}, g^{TN}) = D^{TN}(w^{TN} - c) - \frac{1}{2}\eta g^{TN^2} \quad (2)$$

$$\pi_R^{TN}(p^{TN}) = D^{TN}(p^{TN} - w^{TN}) \quad (3)$$

We solve the game by backward induction. First, solving second-order derivative of equation (3) we can get  $\frac{\partial^2 \pi_R^{TN}}{\partial p^{TN^2}} = -2b < 0$ , so  $\pi_R^{TN}(p^{TN})$  is a strictly concave function of  $p^{TN}$ . Solving first-order condition  $\frac{\partial \pi_R^{TN}}{\partial p^{TN}} = 0$ . Then, substituting  $p^{TN}$  into equation (2), the Hessian matrix is

$$\begin{bmatrix} -b & \frac{k}{2} \\ \frac{k}{2} & -\eta \end{bmatrix} = \frac{4b\eta - k^2}{4} > 0, \text{ so the matrix is negative definite. Solving first-order condition } \frac{\partial \pi_M^{TN}}{\partial w^{TN}} =$$

0 and  $\frac{\partial \pi_M^{TN}}{\partial g^{TN}} = 0$ , we can obtain the equilibrium solutions of manufacturer:

$$w^{TN*} = \frac{2a\eta + c(b\eta - k^2)}{4b\eta - k^2}, g^{TN*} = \frac{(a - bc)k}{4b\eta - k^2}$$

Substituting  $w^{TN*}$  and  $g^{TN*}$  into  $p^{TN}$ , we can obtain the equilibrium solutions of retailer:

$$p^{TN*} = \frac{3a\eta + c(b\eta - k^2)}{4b\eta - k^2}$$

Substituting  $w^{TN*}$ ,  $g^{TN*}$  and  $p^{TN*}$  into equation (1), (2), (3), we can obtain the optimal demand and profits:

$$D^{TN*} = \frac{b(a-bc)\eta}{4b\eta-k^2}, \pi_M^{TN*} = \frac{(a-bc)^2\eta}{2(4b\eta-k^2)}, \pi_R^{TN*} = \frac{b(a-bc)^2\eta^2}{(4b\eta-k^2)^2}$$

**Proposition 1** Manufacturer's wholesale price and produce green degree, retailer's selling price, demand and profits are increasing in product green effect  $k$ , while are decreasing in product green cost coefficient  $\eta$ .

Proposition 1 shows that when only manufacturer takes green marketing strategy, as product green effect  $k$  increases, indicating that consumers' preference for green products increases, manufacturer will increase green investment in products to improve product green degree, which causes that manufacturer raises wholesale price and retailer raises selling price to obtain more benefits. With product green cost coefficient  $\eta$  increasing, it is likely to increase product green cost, so manufacturer will decrease green investment to balance product green cost, resulting in the decrease in product green degree and the market demand. Manufacturer and retailer have to lower the wholesale price and selling price to maintain demand.

### 3.2. Only Retailer Takes Green Marketing (Case NT)

When retailer makes sales effort but manufacturer does not make green investment in products, the market demand also will be reduced but the manufacturer also can benefit from the retailer's green marketing strategy and it does not need to pay green investment cost.

Considering a two-stage Stackelberg game, firstly, the manufacturer decides wholesale price  $w^{NT}$  based on maximizing profit; secondly, the retailer decides simultaneously selling price  $p^{NT}$  and sales effort  $s^{NT}$  based on the manufacturer's optimal decision. The market demand is

$$D^{NT} = a - bp^{NT} + ls^{NT} \quad (4)$$

The profit function of manufacturer and retailer respectively are

$$\pi_M^{NT}(w^{NT}) = D^{NT}(w^{NT} - c) \quad (5)$$

$$\pi_R^{NT}(p^{NT}, s^{NT}) = D^{NT}(p^{NT} - w^{NT}) - \frac{1}{2}\theta s^{NT^2} \quad (6)$$

We solve the game by backward induction. First, the Hessian matrix about equation (6) is  $\begin{bmatrix} -2b & l \\ l & -\theta \end{bmatrix} = 2b\theta - l^2 > 0$ , so the matrix is negative definite. Solving first-order condition  $\frac{\partial \pi_R^{NT}}{\partial p^{NT}} = 0$  and  $\frac{\partial \pi_R^{NT}}{\partial s^{NT}} = 0$ . Then, substituting  $p^{NT}$  and  $s^{NT}$  into equation (5), solving second-order derivative we can get  $\frac{\partial^2 \pi_M^{NT}}{\partial w^{NT^2}} = \frac{2b^2\theta}{l^2 - 2b\theta} < 0$ , so  $\pi_M^{NT}(w^{NT})$  is a strictly concave function of  $w^{NT}$ . Solving first-order condition  $\frac{\partial \pi_M^{NT}}{\partial w^{NT}} = 0$ , we can obtain the equilibrium solutions of manufacturer:

$$w^{NT*} = \frac{a + bc}{2b}$$

Substituting  $w^{NT*}$  into  $p^{NT}$  and  $s^{NT}$ , we can obtain the equilibrium solutions of retailer:

$$p^{NT*} = \frac{b(3a+bc)\theta - (a+bc)l^2}{2b(2b\theta - l^2)}, s^{NT*} = \frac{(a-bc)l}{2(2b\theta - l^2)}$$

Substituting  $w^{NT*}$ ,  $p^{NT*}$  and  $s^{NT*}$  into equation (4), (5), (6), we can obtain the optimal demand and profits:

$$D^{NT*} = \frac{b(a-bc)\theta}{2(2b\theta-l^2)}, \pi_M^{NT*} = \frac{(a-bc)^2\theta}{4(2b\theta-l^2)}, \pi_R^{NT*} = \frac{(a-bc)^2\theta}{8(2b\theta-l^2)}$$

**Proposition 2** Retailer's selling price and sales effort, demand and profits are increasing in sales effort effect  $l$ , while are decreasing in sales effort cost coefficient  $\theta$ .

Proposition 2 shows that when only retailer takes green marketing strategy, manufacturer's wholesale price is independent of sales effort effect  $l$  and effort cost coefficient  $\theta$ . As sales effort effect  $l$  increases, the market demand will increase, even if retailer raises selling price, the demand will not be affected. But meanwhile, it can increase the profit of retailer and manufacturer. With sales effort cost coefficient  $\theta$  increasing, retailer will make less sales effort to balance sales effort cost, which leads to the decrease in the market demand, so the profit of retailer and manufacturer also decrease.

### 3.3. Manufacturer and Retailer Take Green Marketing (Case TT)

When manufacturer makes green investment in products and retailer also makes sales effort, the market demand will be positively affected by their green marketing strategies, leading to maximum market demand, but meanwhile the manufacturer and the retailer need to pay their green marketing cost respectively.

Considering a two-stage Stackelberg game, firstly, the manufacturer decides wholesale price  $w^{TT}$  and product green degree  $g^{TT}$ ; secondly, the retailer decides simultaneously selling price  $p^{TT}$  and sales effort  $s^{TT}$  based on the manufacturer's optimal decision. The market demand is

$$D^{TT} = a - bp^{TT} + kg^{TT} + ls^{TT} \quad (7)$$

The profit function of manufacturer and retailer respectively are

$$\pi_M^{TT}(w^{TT}, g^{TT}) = D^{TT}(w^{TT} - c) - \frac{1}{2}\eta g^{TT^2} \quad (8)$$

$$\pi_R^{TT}(p^{TT}, s^{TT}) = D^{TT}(p^{TT} - w^{TT}) - \frac{1}{2}\theta s^{TT^2} \quad (9)$$

We solve the game by backward induction. First, the Hessian matrix about equation (9) is  $\begin{bmatrix} -2b & l \\ l & -\theta \end{bmatrix} = 2b\theta - l^2 > 0$ , so the matrix is negative definite. Solving first-order condition

$\frac{\partial \pi_R^{TT}}{\partial p^{TT}} = 0$  and  $\frac{\partial \pi_R^{TT}}{\partial s^{TT}} = 0$ . Then, substituting  $p^{TT}$  and  $s^{TT}$  into equation (8), the Hessian matrix is

$$\begin{bmatrix} \frac{2b^2\theta}{l^2-2b\theta} & \frac{-bk\theta}{l^2-2b\theta} \\ \frac{-bk\theta}{l^2-2b\theta} & -\eta \end{bmatrix} = \frac{b^2\theta((4b\eta-k^2)\theta-2l^2\eta)}{(l^2-2b\theta)^2} > 0, \text{ so the matrix is negative definite. Solving first-order}$$

condition  $\frac{\partial \pi_M^{TT}}{\partial w^{TT}} = 0$  and  $\frac{\partial \pi_M^{TT}}{\partial g^{TT}} = 0$ , we can obtain the equilibrium solutions of manufacturer:

$$w^{TT*} = \frac{bc(\eta(2b\theta-l^2)-k^2\theta)+a\eta(2b\theta-l^2)}{b((4b\eta-k^2)\theta-2l^2\eta)}, g^{TT*} = \frac{(a-bc)k\theta}{(4b\eta-k^2)\theta-2l^2\eta}$$

Substituting  $w^{TT*}$  and  $g^{TT*}$  into  $p^{TT}$  and  $s^{TT}$ , we can obtain the equilibrium solutions of retailer:

$$p^{TT*} = \frac{b(3a\eta-c(k^2-b\eta))\theta-(a+bc)l^2\eta}{b((4b\eta-k^2)\theta-2l^2\eta)}, s^{TT*} = \frac{(a-bc)l\eta}{(4b\eta-k^2)\theta-2l^2\eta}$$



Substituting  $w^{TT*}$ ,  $g^{TT*}$ ,  $p^{TT*}$  and  $s^{TT*}$  into equation (7), (8), (9), we can obtain the optimal demand and profits:

$$D^{TT*} = \frac{b(a-bc)\eta\theta}{(4b\eta-k^2)\theta-2l^2\eta}, \pi_M^{TT*} = \frac{(a-bc)^2\eta\theta}{2((4b\eta-k^2)\theta-2l^2\eta)}, \pi_R^{TT*} = \frac{(a-bc)^2\eta^2\theta(2b\theta-l^2)}{2((4b\eta-k^2)\theta-2l^2\eta)^2}$$

**Proposition 3** Manufacture's wholesale price and produce green degree, retailer's selling price and sales effort, demand and profits are increasing in product green effect  $k$  and sales effort effect  $l$ , while they are decreasing in product green cost coefficient  $\eta$  and sales effort cost coefficient  $\theta$ .

Proposition 3 shows that when manufacturer and retailer take green marketing strategy, manufacturer increases product green investment and retailer increases sales effort are helpful to increase the market demand, but meanwhile manufacturer and retailer incurs additional costs. In order to avoid damage to their own benefits, manufacturer will increase wholesale price and retailer will increase selling price, thus their profits also increase. With the increasing of product green cost coefficient  $\eta$  and sales effort cost coefficient  $\theta$ , it is possible to increase the product green investment cost of manufacturer and the sales effort cost of retailer, so manufacturer may decrease product green investment and retailer decreases sales effort, which results in the decrease in market demand. Manufacturer has to lower the wholesale price and retailer has to lower the selling price to maintain demand, thus their profits also decrease. We further compare and analyze manufacturer's produce green degree and profit, retailer's sales effort and profit in different cases, and obtain the following propositions:

**Proposition 4** Comparing produce green degree and the profit of manufacturer and retailer under the TN and TT cases:

$$g^{TN*} < g^{TT*}, \pi_M^{TN*} < \pi_M^{TT*}, \pi_R^{TN*} < \pi_R^{TT*}.$$

Proposition 4 shows that when manufacturer makes green investment in products, manufacturer prefers that retailer also can take green marketing to obtain higher profit, and meanwhile manufacturer will increase green investment in products to improve product green degree, which leads to better market response for green products. Therefore, it is more beneficial for retailer to chooses to make sales effort to cater to consumers' green consumption preferences and maximize profit.

**Proposition 5** Comparing sales effort and the profit of manufacturer and retailer under the NT and TT cases:

$$\pi_M^{NT*} < \pi_M^{TT*}, s^{NT*} < s^{TT*}, \pi_R^{NT*} < \pi_R^{TT*}.$$

Proposition 5 shows that when retailer makes sales effort, it is helpful to increase market demand, so manufacturer should choose to take green investment in products to further improve product green degree to gain better profit. Retailer also prefers manufacturer to take green marketing together, which is more profitable for retailer, and retailer will increase more sales effort to further expand market share to obtain higher benefit.

#### 4. Supply Chain Green Marketing Incentive Contract

Through the above comparison and analysis, we can find when manufacturer and retailer both take green marketing strategy, they can obtain higher profit, which implies that manufacturer and retailer expect other party to make green effort together. Therefore, manufacturer and retailer have incentives to drive other party to take green marketing. We will further examine the models for the manufacturer and retailer to develop incentive contracts separately on the basis of case TT, and compare and analyze the equilibrium solution of manufacturer and retailer.

#### 4.1. Manufacturer Creates Sales Effort Cost-sharing Incentive Contract (Case TZ)

We first explore when manufacturer and retailer both take green marketing strategy, manufacturer creates incentive contract for retailer, assuming that consumers' preference for product green degree gradually increases, manufacturer proposes an incentive contract for retailer's sales effort cost sharing. We assume that manufacturer's sales effort cost sharing ratio for retailer's is  $\lambda (0 < \lambda \leq 1)$ . Manufacturer is bounded rational and it is impossible to undertake the whole sales effort cost of retailer, so we think  $\lambda$  should be within a reasonable range, assuming  $0 < \lambda < \frac{\theta(b\eta-k^2)-l^2\eta}{\theta(b\eta-k^2)}$ . The market demand is

$$D^{TZ} = a - bp^{TZ} + kg^{TZ} + ls^{TZ} \quad (10)$$

The profit function of manufacturer and retailer respectively are

$$\pi_M^{TZ}(w^{TZ}, g^{TZ}) = D^{TZ}(w^{TZ} - c) - \frac{1}{2}\eta g^{TZ2} - \frac{1}{2}\lambda\theta s^{TZ2} \quad (11)$$

$$\pi_R^{TS}(p^{TZ}, s^{TZ}) = D^{TZ}(p^{TZ} - w^{TZ}) - \frac{1}{2}(1-\lambda)\theta s^{TZ2} \quad (12)$$

Similarly, first, the Hessian matrix about equation (12) is  $\begin{bmatrix} -2b & l \\ l & \theta(\lambda-1) \end{bmatrix} = 2b\theta(1-\lambda) - l^2 > 0$ , so the matrix is negative definite. Solving first-order condition  $\frac{\partial \pi_R^{TZ}}{\partial p^{TZ}} = 0$  and  $\frac{\partial \pi_R^{TZ}}{\partial s^{TZ}} = 0$ .

The Hessian matrix about equation (11) is  $\begin{bmatrix} \frac{b^2\theta(l^2(2-3\lambda)-4b\theta(\lambda-1)^2)}{(l^2+2b\theta(\lambda-1))^2} & \frac{bk\theta(2b\theta(\lambda-1)^2+l^2(2\lambda-1))}{(l^2+2b\theta(\lambda-1))^2} \\ \frac{bk\theta(2b\theta(\lambda-1)^2+l^2(2\lambda-1))}{(l^2+2b\theta(\lambda-1))^2} & -\eta - \frac{k^2l^2\theta\lambda}{(l^2+2b\theta(\lambda-1))^2} \end{bmatrix} = \frac{b^2\theta((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}{(l^2+2b\theta(\lambda-1))^2} > 0$ , so the matrix is negative definite. Solving first-order condition  $\frac{\partial \pi_M^{TZ}}{\partial w^{TZ}} = 0$  and  $\frac{\partial \pi_M^{TZ}}{\partial g^{TZ}} = 0$ , we can obtain the equilibrium solutions of manufacturer:

$$w^{TZ*} = \frac{bc(l^2\eta-(k^2-2b\eta)\theta(\lambda-1))(\lambda-1)+a\eta(2b\theta(\lambda-1)^2+l^2(2\lambda-1))}{b((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}, g^{TZ*} = \frac{(a-bc)k\theta(1-\lambda)^2}{(4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2)}$$

Substituting  $w^{TZ*}$  and  $g^{TZ*}$  into  $p^{TZ}$  and  $s^{TZ}$ , we can obtain the equilibrium solutions of retailer:

$$p^{TZ*} = \frac{bc(l^2\eta-(k^2-b\eta)\theta(\lambda-1))(\lambda-1)+a\eta(3b\theta(\lambda-1)^2+l^2(2\lambda-1))}{b((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}, s^{TZ*} = \frac{(a-bc)l\eta(1-\lambda)}{(4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2)}$$

Substituting  $w^{TZ*}$ ,  $g^{TZ*}$ ,  $p^{TZ*}$  and  $s^{TZ*}$  into equation (10), (11), (12), we can obtain the optimal demand and profits:

$$D^{TZ*} = \frac{b(a-bc)\eta\theta(1-\lambda)^2}{(4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2)}, \pi_M^{TZ*} = \frac{(a-bc)^2\eta\theta(\lambda-1)^2}{2((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}, \pi_R^{TZ*} = \frac{(a-bc)^2\eta^2\theta(l^2+2b\theta(\lambda-1))(\lambda-1)^3}{2(l^2\eta(2-3\lambda)+(k^2-4b\eta)\theta(-1+\lambda)^2)}$$



**Proposition 6** (i) If  $0 < \lambda < \frac{1}{2}$ , then  $g^{TZ*} > g^{TT*}$ ,  $\pi_M^{TZ*} > \pi_M^{TT*}$ . Otherwise,  $g^{TZ*} \leq g^{TT*}$ ,  $\pi_M^{TZ*} \leq \pi_M^{TT*}$ .

(ii)  $s^{TZ*} > s^{TT*}$ .

Proposition 6 shows that manufacturer can incentive downstream retailer to make more sales effort through Cost-sharing contracts, which is effectively helpful to coordinate the supply chain. But because manufacturer makes incentive contract with the premise of maximizing its own profit, the highest cost sharing ratio provided by the manufacturer to the retailer will not exceed  $\frac{1}{2}$ . When manufacturer takes active to share the sales effort cost of retailer, retailer will increase more sales effort to increase demand. Meanwhile, when manufacturer realizes that retailer makes more sales effort, it is also more willing to increase more green investment to improve product green degree.

#### 4.2. Retailer Creates Incentive Product Green Cost-sharing Contract (Case TL)

Then, we explore retailer creates incentive contract for manufacturer. Although retailer is a follower in the supply chain, it is responsible for selling products directly to consumers, so retailer still can incentive manufacturer to improve product green degree to maintain a green image. We assume that retailer proposes an incentive contract for manufacturer's product green cost sharing and cost sharing ratio is  $\tau$  ( $0 < \tau \leq 1$ ). Similarly, Retailer is bounded rational, so we assume  $0 < \tau < \frac{2\eta(2b\theta-l^2)-k^2\theta}{2\eta(2b\theta-l^2)}$ . The market demand is

$$D^{TL} = a - bp^{TL} + kg^{TL} + ls^{TL} \quad (13)$$

The profit function of manufacturer and retailer respectively are

$$\pi_M^{TL}(w^{TL}, g^{TL}) = D^{TL}(w^{TL} - c) - \frac{1}{2}\eta(1 - \tau)g^{TL2} \quad (14)$$

$$\pi_R^{TL}(p^{TL}, s^{TL}) = D^{TL}(p^{TL} - w^{TL}) - \frac{1}{2}\theta s^{TL2} - \frac{1}{2}\eta\tau g^{TL2} \quad (15)$$

Similarly, first, the Hessian matrix about equation (15) is  $\begin{bmatrix} -2b & l \\ l & -\theta \end{bmatrix} = 2b\theta - l^2 > 0$ , so the matrix is negative definite. Solving first-order condition  $\frac{\partial \pi_R^{TL}}{\partial p^{TL}} = 0$  and  $\frac{\partial \pi_R^{TL}}{\partial s^{TL}} = 0$ . The Hessian matrix about equation (14) is  $\begin{bmatrix} \frac{2b^2\theta}{l^2-2b\theta} & \frac{bk\theta}{2b\theta-l^2} \\ \frac{bk\theta}{2b\theta-l^2} & \eta(\tau-1) \end{bmatrix} = \frac{b^2\theta(2\eta(2b-l^2)(1-\tau)-k^2\theta)}{(l^2-2b\theta)^2} > 0$ , so the matrix is negative definite. Solving first-order condition  $\frac{\partial \pi_M^{TL}}{\partial w^{TL}} = 0$  and  $\frac{\partial \pi_M^{TL}}{\partial g^{TL}} = 0$ , we can obtain the equilibrium solutions:

$$w^{TL*} = \frac{bc(\theta(k^2+2b\eta(\tau-1))+\eta(1-\tau)(l^2-a(2b\theta-l^2)))}{b(\theta(k^2+4b\eta(\tau-1))+2l^2\eta(1-\tau))}, g^{TL*} = \frac{(a-bc)k\theta}{2\eta(2b-l^2)(1-\tau)-k^2\theta}$$

Substituting  $w^{TL*}$  and  $g^{TL*}$  into  $p^{TL}$  and  $s^{TL}$ , we can obtain the equilibrium solutions of retailer:

$$p^{TL*} = \frac{bc(\eta(b\theta-l^2)(1-\tau)-k^2\theta)+a\eta(3b\theta-l^2)(1-\tau)}{b(2\eta(2b-l^2)(1-\tau)-k^2\theta)}, s^{TL*} = \frac{(a-bc)l\eta(1-\tau)}{2\eta(2b-l^2)(1-\tau)-k^2\theta}$$

Substituting  $w^{TL*}$ ,  $g^{TL*}$ ,  $p^{TL*}$  and  $s^{TL*}$  into equation (13), (14), (15), we can obtain the optimal demand and profits:

$$D^{TL*} = \frac{b(a-bc)\eta\theta(1-\tau)}{2\eta(2b-l^2)(1-\tau)-k^2\theta},$$

$$\pi_M^{TL*} = \frac{(a-bc)^2\eta\theta(1-\tau)}{4\eta(2b\theta-l^2)(1-\tau)-2k^2\theta}, \pi_R^{TL*} = \frac{(a-bc)^2\eta\theta(+\theta(2b\eta(1-\tau)^2-k^2\tau)-l^2\eta(1-\tau)^2)}{2(k^2\theta-2\eta(l^2-2b\theta)(-1+\tau))^2}$$

**Proposition 7** (i)  $s^{TL*} > s^{TT*}$ . If  $0 < \tau < \frac{k^2\theta((4b\eta-k^2)\theta-2l^2\eta)}{\eta(2b\theta-l^2)((8b\eta-k^2)\theta-4l^2\eta)}$ , then  $\pi_R^{TL*} > \pi_R^{TT*}$ . Otherwise,  $\pi_R^{TL*} \leq \pi_R^{TT*}$ .

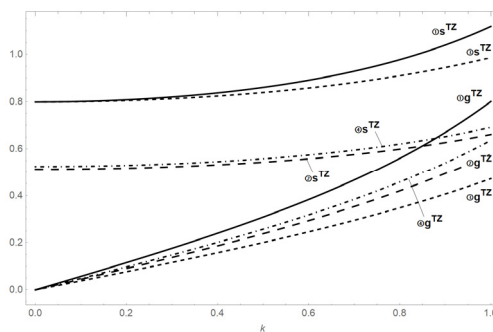
(ii)  $g^{TL*} > g^{TT*}$ ,  $\pi_M^{TL*} > \pi_M^{TT*}$ .

Proposition 7 shows that retailer also can incentive upstream manufacturer to increase green investment through Cost-sharing contracts and retailer can choose suitable cost sharing ration in the optimal range to maximize profit. When retailer shares product green cost with manufacturer, manufacturer will prefer to increase more green investment in products to increase demand and increase profit. When retailer observes that manufacturer is trying to improve product green degree, retailer will also be more active in cooperating with manufacturer to make more sales effort to further increase market demand.

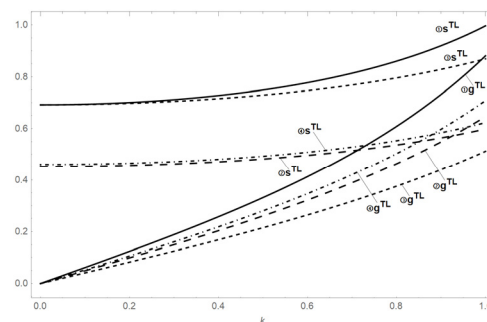
## 5. Numerical Study

In this section, we do numerical study to analyze the impact of important parameters on equilibrium solutions in the incentive scenario and compare the profits of manufacturer and retailer in non-incentive contract and incentive contract. The selection of parameters needs to meet the conditions for the existence of equilibrium solutions.

### 5.1. Sensitivity Analysis in the Incentive Contract



**Figure 1(a).** The impact of  $k, l, \eta, \theta$  on  $g^{TZ}$  and  $s^{TZ}$



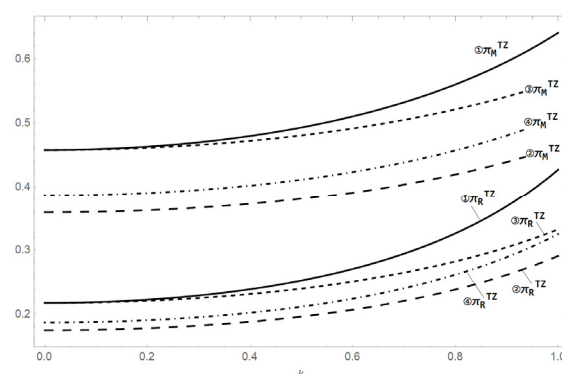
①  $l = 1.1, \eta = 0.8, \theta = 0.7$ , ②  $l = 0.9, \eta = 0.8, \theta = 0.7$   
③  $l = 1.1, \eta = 1.2, \theta = 0.7$ , ④  $l = 1.1, \eta = 0.8, \theta = 0.9$

**Figure 1(b).** The impact of  $k, l, \eta, \theta$  on  $g^{TL}$  and  $s^{TL}$

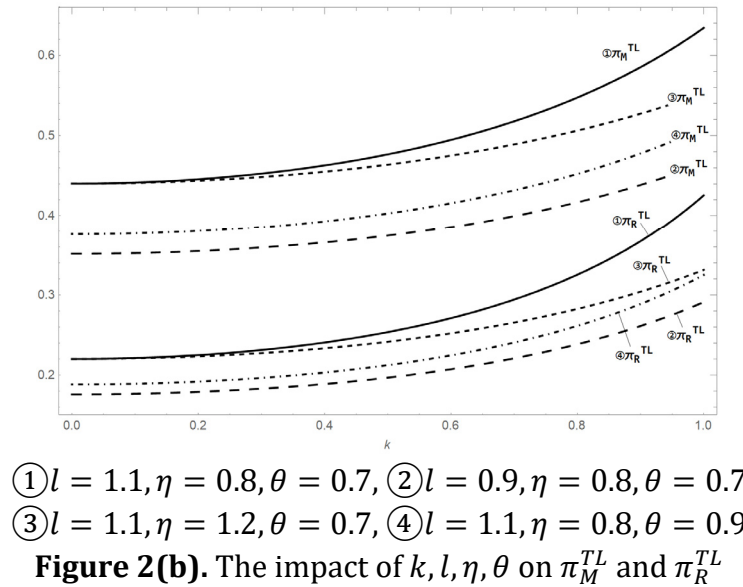
First, we analyze the impact of parameters on product green degree and sales effort in the incentive scenario. The parameters take the value  $a = 4, b = 2, c = 1, \lambda = 0.1, l = \{0.9, 1, 1\}, \eta = \{0.8, 1.2\}, \theta = \{0.7, 0.9\}$ . The numerical experiment results are shown in [Figure 1](#).

From Figure 1 we can find that no matter manufacturer or retailer takes the incentive, as product green effect  $k$  increases, product green degree and sales effort are also increase. Because when consumers' green preference for products increases, it will lead to manufacturer increase green investment in products and meanwhile retailer will be motivated to increase sales effort. We choose  $l = 1.1, \eta = 0.8, \theta = 0.7$  as the control group. We find that when sales effort effect  $l$  decreases ( $l = 1.1 \rightarrow 0.9$ ), consumer insensitivity to sales effort leads retailer to further lower sales effort, which in turn reduce manufacturers' green investment in products. When product green cost coefficient increases ( $\eta = 0.8 \rightarrow 1.2$ ), the increase in product green cost makes manufacturer reduce green investment to balance benefit and retailer has no incentive to increase sales effort. When sales effort cost coefficient increases ( $\theta = 0.7 \rightarrow 0.9$ ), retailer's sales effort cost the increases, which causes retailer reduce sales effort to guarantee its own profit, thus manufacturer reduces green investment in products appropriately, resulting the decrease in product green degree. Besides, we find the impact of product green effect on product green degree changes faster when retailer actively incentivizes manufacturer. Then, we continue to analyze the impact of parameters on the profits of manufacturer and retailer in the incentive scenario. The numerical experiment results are shown in [Figure 2](#).

From Figure.2 we can know that as long as in the incentive contract, when consumers are more sensitive to the green degree level of products, the profit of manufacturer and retailer will also increase. It can be understood that when consumers' green preference for products increases, it will bring higher green investment cost to manufacturer, so the wholesale and selling price of products increase accordingly, and meanwhile with the increase of consumers' green preference, the demand for products will also increase, so the profit of manufacturer and retailer in the supply chain increase. We choose  $l = 1.1, \eta = 0.8, \theta = 0.7$  as the control group. We find that when sales effort effect  $l$  decreases ( $l = 1.1 \rightarrow 0.9$ ), the demand will decrease due to the decrease in sales effort of retailer, which leads that retailer has to lower selling price to keep consumers and manufacturer also has to lower wholesale price to ensure retailer can buy products from it, so the profit of manufacturer and retailer decrease. When product green cost coefficient increases ( $\eta = 0.8 \rightarrow 1.2$ ), the increase in product green cost causes manufacturer's profit decreases, so manufacturer may lower the wholesale price to attract retailer buy more products, but because manufacturer decreases green investment in products, retailer also reduces sales effort, resulting the decrease in market demand, and the profit of retailer decreases. When sales effort cost coefficient increases ( $\theta = 0.7 \rightarrow 0.9$ ), the increase in sales effort cost directly leads retailer to reduce sales effort cost, which causes the demand decreases, and further leads the wholesale price and selling price also decreases, so the profit of manufacturer and retailer will decrease.



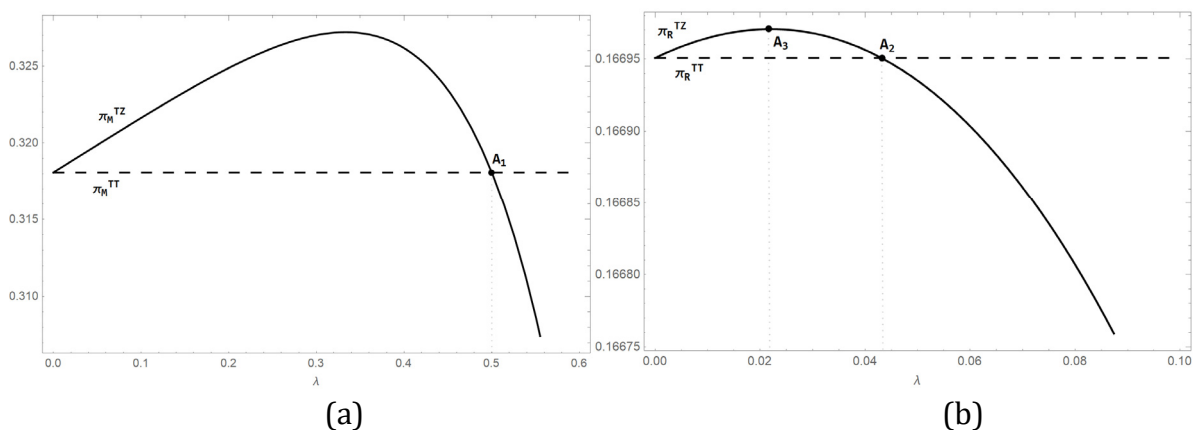
**Figure 2(a).** The impact of  $k, l, \eta, \theta$  on  $\pi_M^{TZ}$  and  $\pi_R^{TZ}$



## 5.2. The Optimal Strategy of Manufacturer When Incentivizing to Retailer

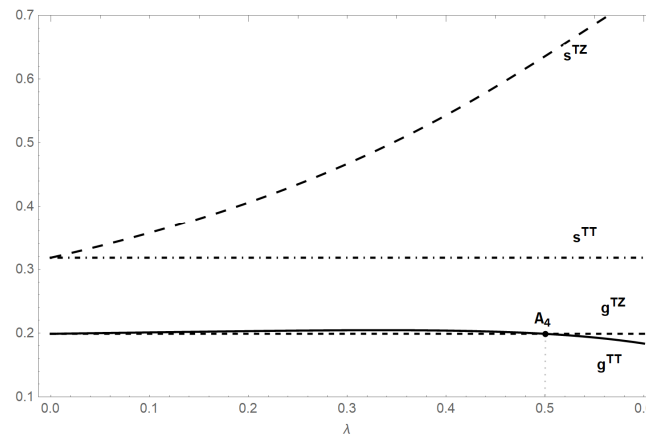
We take  $a = 4, b = 2, c = 1, l = 0.7, \eta = 0.8, \theta = 0.7, k = 0.5$  to compare and analyze the impact of cost sharing ratio on product green degree, sales effort and the profits of manufacturer and retailer when manufacturer takes incentive to retailer. The numerical experiment results are shown in [Figure 3](#). and [Figure 4](#).

From [Figure 3\(a\)](#) we can find that when sales effort cost sharing ratio  $\lambda$  increases, indicating that manufacturer shares a smaller ratio of sales effort cost for retailer, the profit of manufacturer first increase. When sales effort cost sharing ratio increases a threshold, the increase in sharing cost causes that the profit of manufacturer decreases. Besides, we find that when sales effort cost sharing ratio  $\lambda$  is less than a threshold  $A_1$ , the profit of manufacturer with taking incentive is higher than that of without taking incentive, which indicates that manufacturer can choose to share the sales effort cost for retailer to obtain higher profit. Meanwhile, from [Figure 3\(b\)](#) we can observe that when sales effort cost sharing ratio  $\lambda$  is less than a threshold  $A_2$ , the profit of retailer with taking incentive is better than that of without taking incentive. We also find when sales effort cost sharing ratio  $\lambda$  is equal to  $A_3$ , retailer will obtain the highest profit in the incentive contract, and  $A_3 < A_1$ , so manufacturer can set the optimal sales effort cost sharing ratio  $\lambda$  as  $A_3$  to ensure that retailer can make more sales effort actively.



**Figure 3.** The optimal strategy of manufacturer and retailer when manufacturer incentivizes

From Figure 4 we can find that when sales effort cost sharing ratio  $\lambda$  is less than a threshold  $A_4$ , the product green degree in the incentive contract is greater than that of in the non-incentive contract, and  $A_4 = A_1$ , which is consistent with Proposition 4. The sales effort of retailer in the incentive contract is always greater than that of in the non-incentive contract. When retailer observes that manufacturer makes green investment in products and meanwhile shares its sales effort cost, it is helpful to encourage retailer to increase sales effort.

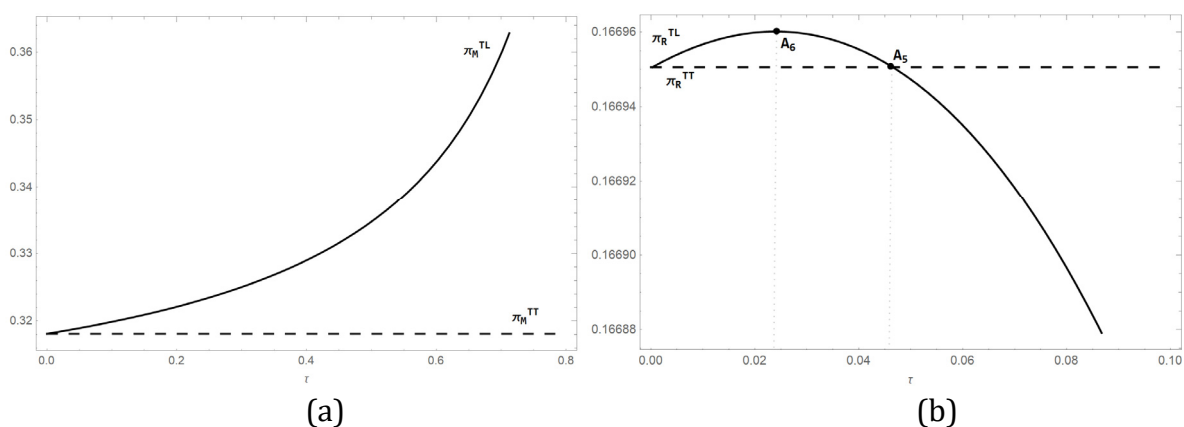


**Figure 4.** Comparison of product green degree and sales effort when manufacturer incentivizes

### 5.3. The Optimal Strategy of Retailer When Incentivizing to Manufacturer

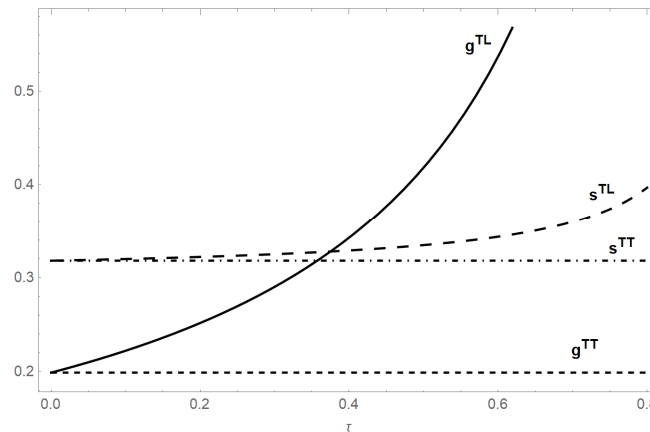
We take  $a = 4, b = 2, c = 1, l = 0.7, \eta = 0.8, \theta = 0.7, k = 0.5$  to compare and analyze the impact of cost sharing ratio on product green degree, sales effort and the profits of manufacturer and retailer when retailer takes incentive to manufacturer. The numerical experiment results are shown in Figure 5. and Figure 6.

From Figure 5(a) we can find that when retailer incentivizes to manufacturer, the profit of manufacturer increases with product green cost sharing ratio  $\tau$  and is always better than without incentive. Because when retailer takes active to share the product green cost for manufacturer, it is beneficial for manufacturer to increase green investment to increase the market demand, further increase profit, while it is not necessarily advantageous for retailer. From Figure 5(b) we can know that when product green cost sharing ratio  $\tau$  is less than a threshold  $A_5$ , the profit of retailer in the incentive contract is higher than that of without incentive, and the retailer's profit reaches a maximum at the critical value of  $A_6$ . Therefore, retailer can set the optimal product green cost sharing ratio as  $A_6$  to obtain maximum profit.



**Figure 5.** The optimal strategy of manufacturer and retailer when retailer incentivizes

From Figure 6 we can find that when retailer incentivizes manufacturer, the product green degree of manufacturer in the incentive contract is always greater than that of in the non-incentive contract, which is easier to understand. When retailer observes that manufacturer tries to improve product green degree, retailer also increases sales effort to further increase the market demand to increase sales revenue and meanwhile can make up for the increase in sharing cost.



**Figure 6.** Comparison of product green degree and sales effort when retailer incentivizes

## 6. Conclusion

In this paper, we study the supply chain composed of a single manufacturer and a single retailer, and explore whether the manufacturer and the retailer take green marketing strategy and how the manufacturer and the retailer establish incentive contract respectively when they take green marketing strategy. We compare and analyze the optimal decisions and profit of manufacturer and retailer, and finally do numerical analysis to further supplement.

The results show that firstly, manufacturer's product green degree and wholesale price, retailer's sales effort and selling price, market demand and profit of manufacturer and retailer are all positively related to product green effect and sales effort effect, while are negatively related to product green cost coefficient and sales effort cost coefficient. Secondly, when only retailer takes green marketing strategy, it is better for manufacturer to choose to make green investment in products. When only manufacturer takes green marketing strategy, it is greater for retailer to chooses to make sales effort. Compared with only one member taking green marketing strategy, it is more advantageous for both retailer and manufacturer to take green marketing strategy. Finally, Cost-sharing contracts can effectively achieve supply chain coordination within the range of cost sharing ratio that manufacturer and retailer are willing to accept, which is more beneficial for manufacturer and retailer to incentivize other partner to improve product green degree and increase sales effort and thus realize profit maximization. Therefore, when developing green products with environmentally friendly attributes, it is necessary to pay attention to upstream and downstream enterprises to coordinate green investment and sales effort to reduce the cost of green practice for both members, and choose an appropriate Cost-sharing contract to achieve the optimal economy of members in supply chain and product green degree maximization.

This paper only considers a two-echelon forward supply chain, and future research can consider the recycling behavior of manufacturer and explore the impact of recycling effort and remanufactured products on closed-loop supply chains. In addition, we can also consider the situation of supply uncertainty and demand uncertainty.



## References

- [1] Liu Y, Zhu Q, Seuring S. Linking capabilities to green operations strategies: The moderating role of corporate environmental proactivity[J]. International Journal of Production Economics, 2017, 187: 182-195.
- [2] Zhang C T, Liu L P. Research on coordination mechanism in three-level green supply chain under non-cooperative game[J]. Applied Mathematical Modelling, 2013, 37(5): 3369-3379.
- [3] Basiri Z, Heydari J. A mathematical model for green supply chain coordination with substitutable products[J]. Journal of cleaner production, 2017, 145: 232-249.
- [4] Ghosh D, Shah J. A comparative analysis of greening policies across supply chain structures[J]. International Journal of Production Economics, 2012, 135(2): 568-583.
- [5] Gao J, Han H, Hou L, et al. Pricing and effort decisions in a closed-loop supply chain under different channel power structures[J]. Journal of Cleaner Production, 2016, 112: 2043-2057.
- [6] Ma P, Li K W, Wang Z J. Pricing decisions in closed-loop supply chains with marketing effort and fairness concerns[J]. International Journal of Production Research, 2017, 55(22): 6710-6731.
- [7] Zerang E S, Taleizadeh A A, Razmi J. Analytical comparisons in a three-echelon closed-loop supply chain with price and marketing effort-dependent demand: game theory approaches[J]. Environment, development and sustainability, 2018, 20(1): 451-478.
- [8] Saha S, Modak N M, Panda S, et al. Promotional coordination mechanisms with demand dependent on price and sales efforts[J]. Journal of Industrial and Production Engineering, 2019, 36(1): 13-31.
- [9] Taleizadeh A A, Zerang E S, Choi T M. The effect of marketing effort on dual-channel closed-loop supply chain systems[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2016, 48(2): 265-276.
- [10] Hong Z, Guo X. Green product supply chain contracts considering environmental responsibilities[J]. Omega, 2019, 83: 155-166.
- [11] Ma P, Wang H, Shang J. Contract design for two-stage supply chain coordination: Integrating manufacturer-quality and retailer-marketing efforts[J]. International journal of production economics, 2013, 146(2): 745-755.
- [12] Taylor T A. Supply chain coordination under channel rebates with sales effort effects[J]. Management science, 2002, 48(8): 992-1007.
- [13] Swami S, Shah J. Channel coordination in green supply chain management[J]. Journal of the operational research society, 2013, 64(3): 336-351.
- [14] Krishnan H, Kapuscinski R, Butz D A. Coordinating contracts for decentralized supply chains with retailer promotional effort[J]. Management science, 2004, 50(1): 48-63.
- [15] Tsao Y C, Sheen G J. Effects of promotion cost sharing policy with the sales learning curve on supply chain coordination[J]. Computers & Operations Research, 2012, 39(8): 1872-1878.
- [16] Li C F, Hu P P, Liu H X. Consumer's green preference, big data targeted advertising and evolution of mobile phone green marketing strategies[J]. Computer Integrated Manufacturing Systems, 2021, 1-18.

## Appendix

**Proof of Proposition 1.** Solving first-order conditions ,  $\frac{\partial w^{TN*}}{\partial \eta} = -\frac{2(a-bc)k^2}{(k^2-4b\eta)^2} < 0$ ,  $\frac{\partial p^{TN*}}{\partial \eta} = -\frac{3(a-bc)k^2}{(k^2-4b\eta)^2} < 0$ ,  $\frac{\partial g^{TN*}}{\partial \eta} = -\frac{4b(a-bc)k}{(k^2-4b\eta)^2} < 0$ ,  $\frac{\partial D^{TN*}}{\partial \eta} = -\frac{b(a-bc)k^2}{(k^2-4b\eta)^2} < 0$ ,  $\frac{\partial \pi_M^{TN*}}{\partial \eta} = -\frac{(a-bc)^2 k^2}{2(k^2-4b\eta)^2} < 0$ ,  $\frac{\partial \pi_R^{TN*}}{\partial \eta} = \frac{2b(a-bc)^2 k^2 \eta}{(k^2-4b\eta)^3} < 0$ .  $\frac{\partial w^{TN*}}{\partial k} = \frac{4(a-bc)k\eta}{(k^2-4b\eta)^2} > 0$ ,  $\frac{\partial p^{TN*}}{\partial k} = \frac{6(a-bc)k\eta}{(k^2-4b\eta)^2} > 0$ ,  $\frac{\partial g^{TN*}}{\partial k} = \frac{(a-bc)(k^2+4b\eta)}{(k^2-4b\eta)^2} > 0$ ,  $\frac{\partial D^{TN*}}{\partial k} = \frac{2b(a-bc)k\eta}{(k^2-4b\eta)^2} > 0$ ,  $\frac{\partial \pi_M^{TN*}}{\partial k} = \frac{(a-bc)^2 k\eta}{(k^2-4b\eta)^2} > 0$ ,  $\frac{\partial \pi_R^{TN*}}{\partial k} = \frac{4bk(a\eta-bc\eta)^2}{(4b\eta-k^2)^3} > 0$ .



**Proof of Proposition 2.** Solving first-order conditions ,  $\frac{\partial p^{NT*}}{\partial \theta} = -\frac{(a-bc)l^2}{2(l^2-2b\theta)^2} < 0$ ,  $\frac{\partial s^{NT*}}{\partial \theta} = -\frac{b(a-bc)l}{(l^2-2b\theta)^2} < 0$ ,  $\frac{\partial D^{NT*}}{\partial \theta} = -\frac{b(a-bc)l^2}{2(l^2-2b\theta)^2} < 0$ ,  $\frac{\partial \pi_M^{NT*}}{\partial \theta} = -\frac{(a-bc)^2 l^2}{4(l^2-2b\theta)^2} < 0$ ,  $\frac{\partial \pi_R^{NT*}}{\partial \theta} = -\frac{(a-bc)^2 l^2}{8(l^2-2b\theta)^2} < 0$ .  $\frac{\partial p^{NT*}}{\partial l} = \frac{(a-bc)l\theta}{(l^2-2b\theta)^2} > 0$ ,  $\frac{\partial s^{NT*}}{\partial l} = \frac{(a-bc)(l^2+2b\theta)}{2(l^2-2b\theta)^2} > 0$ ,  $\frac{\partial D^{NT*}}{\partial l} = \frac{b(a-bc)l\theta}{(l^2-2b\theta)^2} > 0$ ,  $\frac{\partial \pi_M^{NT*}}{\partial l} = \frac{(a-bc)^2 l\theta}{2(l^2-2b\theta)^2} > 0$ ,  $\frac{\partial \pi_R^{NT*}}{\partial l} = \frac{(a-bc)^2 l\theta}{4(l^2-2b\theta)^2} > 0$ .

**Proof of Proposition 3.** Solving first-order conditions, we can get  $\frac{\partial w^{TT*}}{\partial \eta} = \frac{(a-bc)k^2\theta(l^2-2b\theta)}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial p^{TT*}}{\partial \eta} = \frac{(a-bc)k^2\theta(l^2-3b\theta)}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial g^{TT*}}{\partial \eta} = \frac{2(a-bc)k\theta(l^2-2b\theta)}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial s^{TT*}}{\partial \eta} = -\frac{(a-bc)k^2l\theta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial D^{TT*}}{\partial \eta} = -\frac{b(a-bc)k^2\theta^2}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial \pi_M^{TT*}}{\partial \eta} = -\frac{(a-bc)^2 k^2\theta^2}{2(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial \pi_R^{TT*}}{\partial \eta} = \frac{(a-bc)^2 k^2\theta^2(2b\theta-l^2)}{(2l^2\eta+(k^2-4b\eta)\theta)^3} < 0$ .  $\frac{\partial w^{TT*}}{\partial k} = \frac{2(a-bc)k\eta\theta(2b\theta-l^2)}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial p^{TT*}}{\partial k} = \frac{2(a-bc)k\eta\theta(3b\theta-l^2)}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial g^{TT*}}{\partial k} = \frac{(a-bc)\theta(-2l^2\eta+(k^2+4b\eta)\theta)}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial s^{TT*}}{\partial k} = \frac{2(a-bc)kl\eta\theta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial D^{TT*}}{\partial k} = \frac{2b(a-bc)k\eta\theta^2}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial \pi_M^{TT*}}{\partial k} = \frac{(a-bc)^2 k\eta\theta^2}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial \pi_R^{TT*}}{\partial k} = \frac{2(a-bc)^2 k\eta^2\theta^2(l^2-2b\theta)}{(2l^2\eta+(k^2-4b\eta)\theta)^3} > 0$ .  $\frac{\partial w^{TT*}}{\partial \theta} = -\frac{(a-bc)k^2 l^2 \eta}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial p^{TT*}}{\partial \theta} = -\frac{(a-bc)l^2\eta(k^2+2b\eta)}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial g^{TT*}}{\partial \theta} = -\frac{2(a-bc)kl^2\eta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial s^{TT*}}{\partial \theta} = \frac{(a-bc)l\eta(k^2-4b\eta)}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial D^{TT*}}{\partial \theta} = -\frac{2b(a-bc)l^2\eta^2}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial \pi_M^{TT*}}{\partial \theta} = -\frac{(a-bc)^2 l^2 \eta^2}{(2l^2\eta+(k^2-4b\eta)\theta)^2} < 0$ ,  $\frac{\partial \pi_R^{TT*}}{\partial \theta} = \frac{(a-bc)^2 l^2 \eta^2(-2l^2\eta+l^2(k^2+4b\eta)\theta)}{2(2l^2\eta+(k^2-4b\eta)\theta)^3} < 0$ .  $\frac{\partial w^{TT*}}{\partial l} = \frac{2(a-bc)k^2 l \eta \theta}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial p^{TT*}}{\partial l} = \frac{2(a-bc)l\eta(k^2+2b\eta)\theta}{b(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial g^{TT*}}{\partial l} = \frac{4(a-bc)kl\eta\theta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial s^{TT*}}{\partial l} = \frac{(a-bc)\eta(2l^2\eta-k^2\theta+4b\eta\theta)}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial D^{TT*}}{\partial l} = \frac{4b(a-bc)l\eta^2\theta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial \pi_M^{TT*}}{\partial l} = \frac{2(a-bc)^2 l \eta^2 \theta}{(2l^2\eta+(k^2-4b\eta)\theta)^2} > 0$ ,  $\frac{\partial \pi_R^{TT*}}{\partial l} = -\frac{(a-bc)^2 l \eta^2 \theta(-2l^2\eta+(k^2+4b\eta)\theta)}{(2l^2\eta+(k^2-4b\eta)\theta)^3} > 0$ .

**Proof of Proposition 4.**  $g^{TN*} - g^{TT*} = -\frac{2(a-bc)kl^2\eta}{(k^2-4b\eta)(2l^2\eta+(k^2-4b\eta)\theta)} < 0$ , so  $g^{TN*} < g^{TT*}$ ,  $\pi_M^{TN*} - \pi_M^{TT*} = -\frac{(a-bc)^2 l^2 \eta^2}{(k^2-4b\eta)(2l^2\eta+(k^2-4b\eta)\theta)}$ , so  $\pi_M^{TN*} < \pi_M^{TT*}$ ,  $\pi_R^{TN*} - \pi_R^{TT*} = -\frac{(a-bc)^2 l^2 \eta^2(8b\eta^2(2b\theta-l^2)-k^4\theta)}{2(k^2-4b\eta)^2(2l^2\eta+(k^2-4b\eta)\theta)^2}$ , from the assumption  $b > \frac{l^2\eta+k^2\theta}{\eta\theta}$  we can know  $8b\eta^2(2b\theta-l^2)-k^4\theta > 0$ , so  $\pi_R^{TN*} < \pi_R^{TT*}$ .

**Proof of Proposition 5.**  $\pi_M^{NT*} - \pi_M^{TT*} = -\frac{(a-bc)^2 k^2 \theta^2}{4(l^2-2b\theta)(2l^2\eta+(k^2-4b\eta)\theta)} < 0$ , so  $\pi_M^{NT*} < \pi_M^{TT*}$ ,  $s^{NT*} - s^{TT*} = -\frac{(a-bc)k^2 l \theta}{2(l^2-2b\theta)(2l^2\eta+(k^2-4b\eta)\theta)} < 0$ , so  $s^{NT*} < s^{TT*}$ ,  $\pi_R^{NT*} - \pi_R^{TT*} = \frac{(a-bc)^2 k^2 \theta^2(8b\eta\theta-4l^2\eta-k^2\theta)}{8(l^2-2b\theta)(2l^2\eta+(k^2-4b\eta)\theta)^2}$ , from the assumption  $b > \frac{l^2\eta+k^2\theta}{\eta\theta}$  we can know  $8b\eta\theta-4l^2\eta-k^2\theta > 0$  and  $l^2-2b\theta < 0$ , so  $\pi_R^{NT*} < \pi_R^{TT*}$ .

**Proof of Proposition 6.** (i)  $g^{TZ*} - g^{TT*} = \frac{(a-bc)kl^2\eta\theta\lambda(1-2\lambda)}{((4b\eta-k^2)\theta-2l^2\eta)((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}$ , let  $1-2\lambda=0$ , we can get a threshold  $\lambda = \frac{1}{2}$ , if  $0 < \lambda < \frac{1}{2}$ ,  $g^{TZ*} - g^{TT*} > 0$ . Otherwise,  $g^{TZ*} - g^{TT*} \leq 0$ .  $\pi_M^{TZ*} - \pi_M^{TT*} = \frac{(a-bc)^2 l^2 \eta^2 \theta \lambda (1-2\lambda)}{2((4b\eta-k^2)\theta-2l^2\eta)((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}$ , similarly, if  $0 < \lambda < \frac{1}{2}$ ,  $\pi_M^{TZ*} - \pi_M^{TT*} > 0$ . Otherwise,  $\pi_M^{TZ*} - \pi_M^{TT*} \leq 0$ .

(ii)  $s^{TZ*} - s^{TT*} = \frac{(a-bc)l\eta((4b\eta-k^2)\theta(1-\lambda)-l^2\eta)\lambda}{((4b\eta-k^2)\theta-2l^2\eta)((4b\eta-k^2)\theta(1-\lambda)^2+l^2\eta(3\lambda-2))}$ , from the previous assumption, we can know  $s^{TZ*} - s^{TT*} > 0$ , so we can get  $s^{TZ*} > s^{TT*}$ .

**Proof of Proposition 7.** (i)  $s^{TL*} - s^{TT*} = \frac{(a-bc)k^2 l \eta \theta \tau}{((4b\eta-k^2)\theta-2l^2\eta)(2\eta(2b\theta-l^2)(1-\tau)-k^2\theta)}$ , from the previous assumption, we can know  $s^{TL*} - s^{TT*} > 0$ , so  $s^{TL*} > s^{TT*}$ .

$\pi_R^{TL*} - \pi_R^{TT*} = \frac{(a-bc)^2 k^2 \eta \theta^2 \tau (k^2 \eta \theta (2b\theta-l^2)(2+\tau) - k^4 \theta^2 - 4\eta^2 (l^2-2b\theta)^2 \tau)}{2(2l^2\eta+(k^2-4b\eta)\theta)^2 (k^2 \theta - 2\eta(l^2-2b\theta)(-1+\tau))^2}$ , let  $k^2 \eta \theta (2b\theta-l^2)(2+\tau) - k^4 \theta^2 - 4\eta^2 (l^2-2b\theta)^2 \tau = 0$  we can get a threshold  $\tau = \frac{k^2 \theta ((4b\eta-k^2)\theta-2l^2\eta)}{\eta(2b\theta-l^2)((8b\eta-k^2)\theta-4l^2\eta)}$ , so we can know if  $\tau < \frac{k^2 \theta ((4b\eta-k^2)\theta-2l^2\eta)}{\eta(2b\theta-l^2)((8b\eta-k^2)\theta-4l^2\eta)}$ , then  $\pi_R^{TL*} > \pi_R^{TT*}$ . Otherwise,  $\pi_R^{TL*} \leq \pi_R^{TT*}$ .

(ii)  $g^{TL*} - g^{TT*} = \frac{2\eta(2b\theta-l^2)\tau(a-bc)k\theta}{(2l^2\eta+(k^2-4b\eta)\theta)(-\theta(k^2+4b\eta(-1+\tau))+2l^2\eta(-1+\tau))} > 0$ , so we can get  $g^{TL*} > g^{TT*}$ .  $\pi_M^{TL*} - \pi_M^{TT*} = \frac{(a-bc)^2 k^2 \eta \theta^2 \tau}{2(2l^2\eta+(k^2-4b\eta)\theta)(2\eta(2b\theta-l^2)(1-\tau)-k^2\theta)} > 0$ , so we can get  $\pi_M^{TL*} > \pi_M^{TT*}$ .