

Evaluation of Clean Alternative Marine Fuels: Closeness Coefficient based Nonlinear Programming Method for Interval-valued Intuitionistic Fuzzy Multi-attribute Group Decision Making with Incomplete Preference Information

Jingjing Hao

School of Institute of Logistic Science and Engineering, Shanghai Maritime University,
Shanghai 200000, China

Abstract

With the emission of greenhouse gases and the large-scale exploitation of crude oil, it has caused global warming and the shortage of crude oil in the future. This has prompted the search for alternative energy sources to meet the ship's fuel demand and reduce emissions as a promising way to develop green shipping. However, in real life, there is no set of accurate evaluation criteria to measure the superiority of multiple alternative fuels. Therefore, the application of multi-criteria group decision-making is based on considering the complexity of preferences in different aspects and the incompleteness of information, and establishes a set of evaluation standard systems belonging to alternative fuels for ships. This study will use the Interval-valued intuitionistic fuzzy cluster decision nonlinear programming method based on the closeness coefficient under incomplete preference information, and propose a multi-criteria group decision method for ship alternative energy under the condition of incomplete information. According to the combined method, hydrogen fuel is regarded as the best clean alternative energy, followed by liquefied natural gas (LNG), ammonia, biofuel and methanol. According to the sensitivity analysis, changing the attribute weights of operating costs and CO₂ emission reduction efficiency has a significant impact on the research of five alternative fuels for shipping. The incomplete information studied in this paper prefers alternative energy sources and can also be applied to the study of clean fuels for land and air transportation.

Keywords

Alternative Marine Fuels; Nonlinear Programming; Interval-valued Intuitionistic Fuzzy; Multi-attribute Group Decision Making.

1. Introduction

As ship transportation accounts for 80-90% of global trade, the large quantities of freight and passengers is increasing year by year and the cost of transportation is lower than that of land and air transportation, occupying a leading position in economic developing (1). However, the use of petroleum fuels for ship engines has a huge impact on the environment. The dominant emission from ships, which are sulfur oxides (SO_x), nitrogen oxides (NO_x), carbon monoxide (CO), carbon dioxide (CO₂) and particulate matter (PM), have a significantly negative impact on air quality. International Maritime Organization (IMO)(IMO,2014) states that all ships globally consume 300 million tons of fuel annually. Third IMO Greenhouse Gas Study 2014, annual shipboard NO_x emission on 2012 was 19.002 million tons, SO_x emission was 10.240 million tons, which are 15% and 13% of global NO_x and SO_x emission, respectively, and CO, CO₂ and PM emissions were 936 thousand tons, 938 million tons and 1.402 million tons on 2012, respectively. The presence of these gases will adversely effect on human health, such as lung

cancer, cardiopulmonary deaths, bronchitis and pneumonia, and global warming with sea level rise.

In order to reduce air pollution from ships, the IMO has drafted a number of ship emission control regulations. In 2018, the IMO announced an initial agreement to reduce GHG emissions by 50% by 2050 compared to 2008 emissions. It is key regulation for controlling environmental pollution from shipping is the Maritime Agreement Regarding Oil Pollution (MARPOL) for SO_x, NO_x, GHG and PM emission. In order to cope with this issue, the MARPOL in Annex VI revision recommends limiting the sulphur level from the current 3.5% to 0.05%, which is effective January 1, 2020. Since January 2016, NO_x emission in the global and North American emission control areas have been reduced to Tier II and Tier III, respectively. In addition to the relevant international regulations on ship pollution, technical measures for emission reduction (i.e., scrubbers, LNG and low-sulphur fuel) are also widely studied and discussed. With the depletion of petroleum energy and the degradation of ambient air quality, it is now vital to find alternative energy sources. Among these alternative energy technical measures with low-pollution, such as LNG, hydrogen, electricity and methanol for the propulsion of ships, have been considered as possible pathways for mitigating the high energy consumption and severe environment problems of shipping. LNG does not contain sulphur, meaning that the SO_x emissions are theoretically reduced to zero(1). Compared with HFO, the use of LNG as an alternative fuel for ships reduces NO_x (38-39%) and CO (42-43%) emissions. The reduction of SO₂ (99.8%) and PM₁₀ (97.5%) was more significant(2). Biofuels could help to achieve pollution emissions reduction targets. All biofuels contain very little sulphur and exhibits lower NO_x and PM emissions than marine gas oil(3). Since the raw materials of biodiesel are usually derived from plants that do not contain any sulfur elements, biofuels are considered to be sulfur-free products. The carbon dioxide emitted by ships using biodiesel has decreased, ranging from 0.3% to 3.1%, and NO_x emission are reduced 1.1% to 24.3%(4).

The results of the Bicer and Dincer(5) study show that if ammonia is used as dual fuel (heavy fuel oils) in the engines of ocean tankers, the greenhouse gas emissions per metric ton kilometer during the entire life-cycle can be reduced by about 27%, while hydrogen is used as dual fuel can be reduced by about 40%.

Therefore, the use of alternative energy sources to promote ships has become a hot topic in achieving green and environmentally friendly shipping.

Different alternative energy sources for ships have economic, environmental and policy differences. For instance, an alternative energy source may perform better than others in one aspect, but may perform worse in another aspect(6). Our study focuses on alternative fuels for shipping selection decision which is a typical multicriteria problem entailing to consider a variety of quantitative and qualitative criteria in the fuzzy decision-making process. In classical multicriteria decision making methods, the judgments of decision makers are represented by crisp numbers. However, in the real industrial application, experts generally prefer making linguistic assessments rather than exact numerical judgements. In order to solve the ambiguity, vagueness, subjectivity in the human judgments, the intuitionistic fuzzy set theory was introduced by Zadeh L A(7). In the study, alternative fuels for shipping selection problem are solved under fuzzy environment by considering uncertainties and ambiguities in the decision-making process. Such linguistic assessment can be converted to their corresponding numerical values and then be incorporated into a MCDM method through the fuzzy set theory. Therefore, it is crucial to establish a multi-criteria decision support framework to help decision makers choose the most suitable alternative energy sources according to their preferences and the actual environment.

Compared with the previous application of the MCDM method in alternative energy options for ships, this paper differs in the following three points: (i) in the existing literature, however, there is little research on simultaneously determining the weights of the decision-making

experts and the attributes in the group decision-making problems; (ii) the application of IVIFSs theory provides an intuitive and computationally feasible method to deal with uncertain and partially known attributes; (iii) for the multiple criteria group decision-making (MCGDM) problem, where the information about the criterion weights is completely unknown or incompletely known a priori, two optimization models are constructed to solve the optimal weight values and determine the corresponding inclusion-based closeness coefficients. In the paper, we have developed a method to fill these research gaps. Therefore, the novelties of this study is to develop a multi-criteria group decision making method by IVIFSs theory, the non-linear programming (NLP) methodology and the extension of the technique for TOPSIS method, which the NLP methodology is used to obtain optimal weights of attributes, and allows the decision-makers to use linguistic terms to express their opinion on the relative importance of the criteria for selecting the most sustainable alternative energy sources for shipping.

2. Literature Review of MCDM Method and Fuzzy Set Theory

In the literature, the frequently used method for alternative fuels are AHP (Analytic Hierarchy Process), ANP (Analytic Network Process), TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje), ELECTRE (Elimination and Choice Translating Reality). The crisp applications of these methods can not employ the vague evaluations which humans generally prefer, while their fuzzy sets extensions can do it. Therefore, ordinary fuzzy sets have been widely used in MCDM methods such as fuzzy TOPSIS, fuzzy AHP, and fuzzy VIKOR. Extensions of ordinary fuzzy sets, such as hesitant fuzzy sets, intuitionistic fuzzy sets, and type-2 fuzzy sets have been introduced into multi-criteria decision-making and shown the advantage of better definition of membership function than ordinary fuzzy sets(8). Intuitionistic fuzzy sets extend the ordinary fuzzy sets by an additional degree, which is called the degree of non-membership. Atanassov(1986) proposed the theory of intuitionistic fuzzy sets (IFs) in 1986. An IFs is associated with the membership function, the non-membership function and the hesitancy function. Atanassov and Gargov(1989) proposed the theory of Interval-valued intuitionistic fuzzy sets (IVIFs) in 1989, which is an extension of the theory of the theory of IFs. IVIFs are represented by an Interval-valued membership degree and an Interval-valued non-membership degree. According to some authors, IVIFs is more powerful and flexible tool to cope with vagueness and uncertainty than the other types of IFs.

IVIF-TOPSIS is integrated with different techniques in various fields of application. On the other hand, IVIF-TOPSIS is utilized with the concept of inclusion comparison possibilities which is different from convention closeness coefficient for ranking the alternatives. To author's best knowledge, there exists no publication in which IVIF-TOPSIS based on the concept of inclusion comparison possibilities is used for sustainable clean energy selection for shipping. Therefore, this paper contributes to the literature by addressing this research gap and demonstrating the applicability of the proposed method with alternative fuel selection. As seen in [Table 1](#), there are few works on the evaluation of alternative energy technologies using IVIFs MCDM method in the literature.

In this paper, we develop a new TOPSIS model involving an inclusion comparison closeness coefficient approach for solving MCGDM for alternative fuels in shipping selection problems within the Interval-valued intuitionistic fuzzy environment. Furthermore, for MCGDM problems in which the information about the criterion weights is completely unknown or is incompletely known a priori, two optimization models are constructed to solve for the optimal weight values and to determine the corresponding inclusion comparison closeness coefficients.

Table 1. Several studies make use of alternative fuels

Year	Authors	Model	Application area
2015	Onar et al.(9)	IVIFS	Evaluate wind energy investments for wind energy technology selection
2016	Deniz and Zincir (10)	AHP	Assessment of the possibilities for selected alternative fuels for the maritime sector
2017	Oztaysi et al.(8)	IVIFS, TOPSIS, Multiple attribute group decision making (MAGDM)	Alternative-fuel technology selection problem (Alternative-fuel vehicles)
2017	Kumar et al.(11)	MCDM review	Sustainable renewable energy development
2017	Ren and Liang (6)	fuzzy logarithmic least squares and fuzzy TOPSIS	Study measuring the sustainability of marine fuels
2017	Ren and Lützen (6)	Dempster-Shafer theory and a trapezoidal fuzzy analytic hierarchy process	Alternative energy selection under incomplete information conditions
2017	Svanberg et al. (12)	MCDM and AHP	Assessment of the possibilities for selected alternative fuels for the maritime sector
2017	Hua et al.(2)	Total fuel life-cycle inventory	Alternative fuel for sustainable shipping across the Taiwan Strait
2018	Gilbert et al.(13)	life-cycle	A life-cycle assessment with respect to six emissions species
2021	Prussi et al.(14)	“Fleets and Fuels” (FF20) modeling	Potential and limiting factors in the use of alternative fuels in the European maritime sector
2021	Al-Enazi et al.(15)	review	Clean alternative fuels for maritime transportation

3. Sustainability Assessment of Alternative Energy Sources

Alternative fuels which can be used at marine diesel engines are found in two types: liquid fuels like bioliquid fuel, biodiesel, methanol (CH₃OH) and ethanol (C₂H₅OH); and gaseous fuels like propane, hydrogen and natural gas(10). Alternative fuels for shipping include any sustainable alternative fuels that are suitable for providing marine transportation, potentially offering environmental benefits when compared against traditional diesel fuels. In the following Section 3.1, we focus on five types of alternative fuels and briefly state some of their key characteristics (See [Table 2](#)).

3.1. Alternative Fuels for Shipping

3.1.1. LNG

LNG is a transparent, odorless, non-toxic, non-corrosive at atmospheric pressure cryogenic liquid. Natural gas is actually a fossil fuel. However, it reduces carbon emissions and has more common characteristics with other non-traditional fuels, so it is considered an alternative fuel. There are two forms of natural gas (enriched and liquefied) as alternative fuels, of which liquefied natural gas (LNG) is the more commonly used form (8). In maritime transportation, LNG is the only feasible and mature technology. In the field of heavy-duty and long-distance transportation, LNG is an alternative to diesel (30).

3.1.2. Methanol

Methanol, also known as methanol alcohol or wood alcohol, is a simple alcohol that burns cleanly(12). The main raw materials of methanol are natural gas and coal, but it can also be produced from renewable raw materials. In the production of bio-methanol, primary raw materials (direct sources) and secondary raw materials (by-products) can be used. Shamsul et al.(16) acknowledged that there are multiple sources of raw materials that may be used for methanol production, such as agricultural and forest residues, livestock and poultry waste, fishery waste, and sewage sludge.

3.1.3. Hydrogen

Hydrogen is the most abundant element on earth, but it is usually found in the more stable form of water, less than 1% of the gas is easily available(5). The required hydrogen can be obtained in various ways, one of the cleanest methods is electrolysis. The electrolysis process uses electricity to split water into hydrogen and oxygen(8). In addition to fossil fuels, hydrogen can also be produced from the conversion of biogas and renewable methane from electrolyzed water using renewable energy such as wind, solar and water power(17).

3.1.4. Biodiesel

Biodiesel is essentially a renewable diesel, which can be made from a variety of raw materials. In Europe and the United States, rapeseed oil and soybean oil are commonly used to produce biodiesel. Tropical countries including Malaysia, Thailand, Indonesia, Nigeria and Colombia extract biodiesel from palm oil(21). It is very important to choose the right feedstock to produce biodiesel because it is related to 75% of the total cost(22). In addition, the quality of biodiesel also depends on the type of resource use, production process and country of origin(23). Generally speaking, the sources of biodiesel can be divided into four categories: edible vegetable oil, non-edible vegetable oil, recycle and waste oil and animal fats.

3.1.5. Ammonia

Ammonia (NH₃) produced from hydrogen and nitrogen may have less climate impact on renewable energy (or combined with carbon capture and storage (CCS)). However, currently ammonia is mainly produced by fossil fuel-based hydrogen while production process of renewable ammonia is still under development(24). Ammonia has been proven to be the fuel for compression ignition (CI) engines, spark ignition (SI) engines and fuel cells.

Table 2. Alternative fuels characteristics

Year	LNG	Methanol	Hydrogen	Biodiesel	Ammonia
Density (kg/m ³)	400-500	798	0.0838	860-900	0.771
Auto-ignition temperature (K at 1 bar)	810	743	858	100-170	650-657
Net heating value (MJ/kg)	46-50.2	20.1	119.9	35	18.6
Cetane number	-10	<5	-	51	-
Fuel carbon content (wt %)	75	38	0	77	0
Fuel hydrogen content (wt %)	25	12	100	15	17.8
Fuel oxygen content (wt %)	0	50	0	6	0
Fuel sulfur content (wt %)	1	0	0	0.05	0
Toxic	No	No	No	No	Yes
References	(21),(10), (20),(25),(24)				

3.2. Criteria for Alternative Energy Sources

The sustainable development of alternative fuels can be interpreted in many different perspectives. We adopt a triple bottom line approach and the criteria used for sustainability assessment are defined within the three aspects of sustainability to cover economic, environmental and social concerns. M.Prussi et al.(14), study the vast majority of the available literature focus on the cost differential for the alternative fuels relative to HFO and diesel, and the potential environmental benefits of the proposed solutions. There are a few published works concerning alternative fuel for shipping using IVIF method to evaluate and select. We refer the reader to Ren and Lützen(6) or Ren and Liang(26) for a survey of criteria adopted in the literature. The present study uses four dimensions, which are the technological, economic, environmental and Social-political aspects to measure sustainability

The criteria system for sustainability assessment was developed for sustainability assessment of marine fuels. For the sustainability evaluation of alternative energy sources for shipping, the criteria system including twelve indicators in four aspects (see Table 3). There are three criteria in the technological aspect, including maturity, reliability and energy storage efficient; the economic aspect consists of investment cost and operation cost; effect on CO₂ emission reduction, effect on NO_x emission reduction, effect on SO_x emission reduction and on PM emission reduction are the four criteria belonging to the environmental aspect; and finally safety, social acceptability and governmental support are the three criteria of the social-political.

Table 3. Criteria for sustainability assessment of marine fuels

Category	Criteria	Abbr	references
Technological (T)	Maturity	T1	(25)
	Reliability	T2	(10)
	Capacity	T3	(10; 6)
Economic (EC)	Investment cost	EC1	(10; 6; 12)
	Operation cost	EC2	(10; 6)
Environmental (EN)	Effect on CO ₂ emission reduction	EN1	(21)
	Effect on NO _x emission reduction	EN2	(35; 37)
	Effect on SO _x emission reduction	EN3	(21)
	Effect on PM emission reduction	EN4	(36)
Social-political (SP)	Safety	SP1	(10; 6)
	Social acceptability	SP2	(6)
	Governmental support	SP3	(6)

The alternative energy sources within shipping primarily refer to LNG, methanol, hydrogen, electricity, biodiesel and ammonia, and the stakeholders usually consider the above-mentioned criteria to select the most suitable option when facing multiple alternative energy sources. These criteria are specified as follows:

3.2.1. Technological

Maturity

The market maturity of alternative fuels is a measure of technology maturity, which reflects the feasibility and popularity of different fuels in the world. LNG fuel has been used in steam turbines and dual-fuel diesel engines in the 1970s. It has been developed for 50 years and has been widely used in shipping industry fuels (25). In addition, the global LNG trade volume increased from 100 million tons in 2000 to 319 million tons in 2018 (31).

Reliability

The reliability of alternative fuels mainly considers the sustainability of ship fuel use. Different fuels can cause different degrees of damage to ship engines. This standard is used to measure the degree of impact on engine performance(10). Global production of ammonia in 2016 was approximately 180 million tons(24)

Capacity

The engine fuel tank capacity is related to global availability and is also closely related to the fuel performance of the fuel(10; 6). Methanol fuel can be stored in the ship's original gasoline storage tank, and the storage space is small for modification(19). Liquefied natural gas (LNG), as a cryogenic liquid, must be stored in a separate pressure tank, which is disadvantage for volume critical ships(12).

3.2.2. Economic

Investment cost

Infrastructure facilities refer to supply bases built around the world for alternative fuels to facilitate the navigation of different ships. For instance, many ports around the world, such as Rotterdam and Antwerp in Europe, already have dedicated methanol storage and supply infrastructure(32). Only minor modifications of the infrastructure are needed to use methanol as a marine fuel, especially when compared to the implementation of LNG infrastructure.

Capital expenditure refers to the cost of retrofitting old ships to adopt new alternative energy sources or investing in the construction of new ships. Shipowners need to be certain about the long-term availability of fuel before investing more money to adjust fuel systems and engines(12). For instance, Biodiesel can be applied to diesel engines without any changes to the engine system because its combustion characteristics are almost similar to conventional diesel(21). Ammonia is corrosive, which needs to be considered in the design of marine fuel systems(17).

Operation cost

Maintenance cost refers to the process of ship using alternative energy sources. Because different fuels have different properties, they will cause different degrees of wear and tear on ship engines. It simply refers to the maintenance and repair costs of infrastructure.

Training costs and crew salaries refer to the personnel training and education costs required for operators to use new alternative energy sources. Because hydrogen has a higher natural temperature than other fuels, it is prone to explosion. The storage requirements are high, requiring specially trained crew to keep and operate correctly(5).

Fuel price refers to the price of alternative energy for ships. Fuel price is a key factor affecting the total cost of ship operations. On the one hand, depending on the fuel characteristics, hydrogen liquefaction requires a very low temperature, namely 253°C, which causes high costs for the ship's liquefaction and storage system. On the other hand, ammonia gas has a high volumetric energy density and easy handling, which is a hot topic for the shipping industry to achieve decarbonization goals(20). However, the volume density of liquefied hydrogen is lower than that of HFO, and the price of hydrogen is about 2.7 to 3.5 times that of HFO(19).

3.2.3. Environmental

Effect on CO₂ emission reduction

Carbon dioxide is a greenhouse gas, which produces the greenhouse effect by absorbing infrared radiation. However, too much of these gases will absorb heat in the atmosphere, thereby warming the earth(21). For instance, a study by Nick Ash (32) concluded that green ammonia produced using renewable electricity will not emit greenhouse gases at any time during the product life cycle, and it is a technically feasible solution for international shipping to decarbonize.

Effect on NO_x emission reduction

NO_x emission comes from the gas produced by the high-temperature combustion of fuel in the engine cylinder, the temperature of which is as high as 1500°C. By reading the literature, burning hydrogen (not pure hydrogen) with air at a certain temperature will lead to the emission of pollutants NO_x(5).

Effect on SO_x emission reduction

Sulfur oxide can be defined as the type of sulfur and oxygen-containing compounds, such as SO, SO₂, SO₃, S₇O₂, S₆O₂ and S₂O₂(21). Due to the combustion of fuel, this gas is present in engine emissions. Therefore, SO_x is directly proportional to the total sulfur content of the fuel, which can be alleviated by reducing the sulfur content of the fuel. For instance, since biodiesel raw materials are usually derived from plants, which does not contain any sulfur elements, biofuels are regarded as sulfur-free products; therefore, it is believed that the introduction of biodiesel mixtures on ships can significantly reduce the emissions of sulfur compounds(21). As a marine engine fuel, methanol complies with strict international sulfur emission standards regardless of the raw material used to make methanol, because the combustion process does not contain sulfur(12).

Effect on PM emission reduction

Particulate matter (PM) is related to low-quality marine fuel. The combustion process of diesel engines releases harmful particles, also known as soot. The basic components of PM emissions include carbon, heavy hydrocarbons, and hydrated sulfuric acid(36). Past literature has shown that, due to the oxygen and low sulfur content in biodiesel molecules, adding biodiesel to conventional diesel can generally reduce PM emissions(21).

3.2.4. Social-political

Safety

Safety is the primary prerequisite for judging the performance of alternative energy sources. The safety of alternative fuels includes the safety of life and work on board during the operation of the ship, as well as the safety of residents in and around the port(6). Ammonia is a toxic substance. The release of high concentrations of ammonia into the atmosphere can cause health hazards and is fatal within a certain concentration and time period(17). The safety of hydrogen is a problem that requires special attention, because H₂ is explosive, with a flammability range of between 4% and 77% when mixed with air(18).

Social acceptability

Social acceptability refers to the degree to which the selected alternative fuel is acceptable to the public, which can be defined by the breadth of fuel application on ships(6). For instance, the main disadvantages of ammonia are toxicity and environmental impact. Ammonia is toxic if inhaled, and exposure to ammonia can cause severe skin burns and eye damage(19).

Governmental support

Policy support is to measure how the adoption of alternative energy sources for shipping to meet the management standards and policy standards drafted by the government administration (6).

4. Method

4.1. The Concept of the IFs and IVIFs

The concept of the intuitionistic fuzzy sets (IFs) was firstly introduced by Atanassov(27) in 1986 take into account both the membership degree and the non-membership degree for describing any x in X . Let $X = \{x_i \mid i=1,2,\dots,n\}$ denote a finite universal of discourse. The basic definitions of IFs are given in the following parts.

Definition 1. An IFs A in X , where $X \neq \emptyset$ be a given set, can be defined as follows(Atanassov, 1986):

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in X \} \tag{1}$$

where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ satisfy the condition $0 \leq \mu_A + \nu_A \leq 1, \forall x \in X$. μ_A and ν_A denote the membership function and the non-membership function of the element x to the set A , respectively. The pair $\langle \mu_A(x), \nu_A(x) \rangle$ is called intuitionistic fuzzy value (IFV) and simply denoted by $A = \langle \mu_A(x), \nu_A(x) \rangle$. In addition, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic fuzzy index or Hesitancy of an element x in the set A . It is the degree of indeterminacy membership of the element x to the set A . Obviously, $0 \leq \pi_A(x) \leq 1$. Atanassov point out that an IFV $\langle \mu_A(x), \nu_A(x) \rangle$ could be converted into an interval number $[\mu_A(x), 1 - \nu_A(x)]$

Definition 2. Let $X = \{x_i | i = 1, 2, \dots, n\}$ be a finite universal set and I be the set of all closed subintervals of the interval $[0,1]$. An IVIFs \tilde{A} in X is an object having the following form (28):

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle | x \in X \} \tag{2}$$

where $\mu_{\tilde{A}}: X \rightarrow I_{[0,1]}$ and $\nu_{\tilde{A}}: X \rightarrow I_{[0,1]}$ and $0 \leq \sup\{\mu_{\tilde{A}}(x)\} + \sup\{\nu_{\tilde{A}}(x)\} \leq 1$ for every $x \in X$. The intervals $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ represent membership degree and non-membership degree of the element x to the set $\tilde{A} \subseteq X$, respectively. Obviously, $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ are closed intervals.

For the convenience of description, the lower and upper bounds of the interval-value membership degree and interval-value non-membership degree are denoted by $\mu_{\tilde{A}}^l(x), \mu_{\tilde{A}}^u(x), \nu_{\tilde{A}}^l(x)$ and $\nu_{\tilde{A}}^u(x)$, respectively. IVIFs \tilde{A} can be expressed in interval-value form as

$$\tilde{A} = \{ \langle x, [\mu_{\tilde{A}}^l(x), \mu_{\tilde{A}}^u(x)] [\nu_{\tilde{A}}^l(x), \nu_{\tilde{A}}^u(x)] \rangle | x \in X \} \tag{3}$$

where $0 \leq \mu_{\tilde{A}}^l(x) \leq \mu_{\tilde{A}}^u(x) \leq 1, 0 \leq \nu_{\tilde{A}}^l(x) \leq \nu_{\tilde{A}}^u(x) \leq 1$, and $\mu_{\tilde{A}}^u(x) + \nu_{\tilde{A}}^u(x) \leq 1$ for every $x \in X$. The hesitancy degree (fuzzy index) of an IVIF set of $x \in X$ in $\tilde{A} = ([\mu_{\tilde{A}}^l(x), \mu_{\tilde{A}}^u(x)] [\nu_{\tilde{A}}^l(x), \nu_{\tilde{A}}^u(x)])$ is $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) = [1 - \mu_{\tilde{A}}^u(x) - \nu_{\tilde{A}}^u(x), 1 - \mu_{\tilde{A}}^l(x) - \nu_{\tilde{A}}^l(x)]$.

An IVIF set \tilde{A} can be denoted as $\tilde{A} = ([\mu_{\tilde{A}}^l, \mu_{\tilde{A}}^u], [\nu_{\tilde{A}}^l, \nu_{\tilde{A}}^u])$, where $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^l(x), \mu_{\tilde{A}}^u(x)] = [\mu_{\tilde{A}}^l, \mu_{\tilde{A}}^u]$ and $\nu_{\tilde{A}}(x) = [\nu_{\tilde{A}}^l(x), \nu_{\tilde{A}}^u(x)] = [\nu_{\tilde{A}}^l, \nu_{\tilde{A}}^u]$.

Definition 3. Some arithmetic operations on IVIF numbers are gives as follows (29):

Let $\tilde{A} = ([\mu_{\tilde{A}}^l, \mu_{\tilde{A}}^u], [\nu_{\tilde{A}}^l, \nu_{\tilde{A}}^u])$ and $\tilde{B} = ([\mu_{\tilde{B}}^l, \mu_{\tilde{B}}^u], [\nu_{\tilde{B}}^l, \nu_{\tilde{B}}^u])$ be two INIF numbers, and an ordinary $\lambda > 0$. Then,

$$\tilde{A} \oplus \tilde{B} = ([\mu_{\tilde{A}}^l + \mu_{\tilde{B}}^l - \mu_{\tilde{A}}^l \mu_{\tilde{B}}^l, \mu_{\tilde{A}}^u + \mu_{\tilde{B}}^u - \mu_{\tilde{A}}^u \mu_{\tilde{B}}^u], [\nu_{\tilde{A}}^l \nu_{\tilde{B}}^l, \nu_{\tilde{A}}^u \nu_{\tilde{B}}^u]) \tag{4}$$

$$\tilde{A} \otimes \tilde{B} = ([\mu_{\tilde{A}}^l \nu_{\tilde{B}}^l, \mu_{\tilde{A}}^u \nu_{\tilde{B}}^u], [\nu_{\tilde{A}}^l + \nu_{\tilde{B}}^l - \nu_{\tilde{A}}^l \mu_{\tilde{B}}^l, \nu_{\tilde{A}}^u + \nu_{\tilde{B}}^u - \nu_{\tilde{A}}^u \mu_{\tilde{B}}^u]) \tag{5}$$

$$\lambda \cdot \tilde{A} = ([1 - (1 - \mu_{\tilde{A}}^l(x))^\lambda, 1 - (1 - \mu_{\tilde{A}}^u(x))^\lambda], [\nu_{\tilde{A}}^l(x)^\lambda, \nu_{\tilde{A}}^u(x)^\lambda]) \tag{6}$$

$$(\tilde{A})^\lambda = \left(\left[(\mu_{\tilde{A}}^l(x))^\lambda, (\mu_{\tilde{A}}^u(x))^\lambda \right], \left[1 - (1 - v_{\tilde{A}}^l(x))^\lambda, 1 - (1 - v_{\tilde{A}}^u(x))^\lambda \right] \right) \tag{7}$$

Definition 5. Let $\tilde{\alpha}_i = ([a_i, b_i][c_i, d_i])$ ($i = 1, 2, \dots, m$) be a collection of IVIFVs(33).

$$\text{IVIFWM}_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_m) = ([\sum_{i=1}^m w_i a_i, \sum_{i=1}^m w_i b_i], [\sum_{i=1}^m w_i c_i, \sum_{i=1}^m w_i d_i]) \tag{8}$$

where $w = (w_1, w_2, \dots, w_m)^T$ is a weight vector of $\tilde{\alpha}_i$ ($i = 1, 2, \dots, m$), satisfying that $w_i \in [0, 1]$ ($i = 1, 2, \dots, m$) and $\sum_{i=1}^m w_i = 1$, then the IVIFWM is called an IVIF weighted arithmetic mean (IVIFWM) operator of dimension m . Specially, when $w = (1/n, 1/n, \dots, 1/n)^T$, the IVIFWM is called the IVIF arithmetic mean (IVIFM) operator.

Definition 6. Let $\tilde{A} = ([\mu_{\tilde{A}}^l, \mu_{\tilde{A}}^u], [v_{\tilde{A}}^l, v_{\tilde{A}}^u])$ and $\tilde{B} = ([\mu_{\tilde{B}}^l, \mu_{\tilde{B}}^u], [v_{\tilde{B}}^l, v_{\tilde{B}}^u])$ be two IVIFVs. The distance between them is defined as(34):

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4} \left[(\mu_{\tilde{A}}^l - \mu_{\tilde{B}}^l)^2 + (\mu_{\tilde{A}}^u - \mu_{\tilde{B}}^u)^2 + (v_{\tilde{A}}^l - v_{\tilde{B}}^l)^2 + (v_{\tilde{A}}^u - v_{\tilde{B}}^u)^2 \right]} \tag{9}$$

Definition 7. Let $\tilde{R}^k = ([\mu_{ijl}^k, \mu_{iju}^k], [v_{ijl}^k, v_{iju}^k])_{m \times n}$ ($k = 1, 2$) be two IVIF matrices. The distance between \tilde{R}^1 and \tilde{R}^2 is defined as(33)

$$d(\tilde{R}^1, \tilde{R}^2) = \sqrt{\frac{1}{4mn} \sum_{i=1}^m \sum_{j=1}^n \left[(\mu_{ijl}^1 - \mu_{ijl}^2)^2 + (\mu_{iju}^1 - \mu_{iju}^2)^2 + (v_{ijl}^1 - v_{ijl}^2)^2 + (v_{iju}^1 - v_{iju}^2)^2 \right]} \tag{10}$$

4.2. MAGDM Problems with IVIF Sets and Incomplete Preference Information

This paper apply a method for solving multiple attribute group decision-making (MAGDM) problems with Interval-valued intuitionistic fuzzy values and incomplete attribute weight information. Assume that $[\mu_{ijl}^k, \mu_{iju}^k]$ and $[v_{ijl}^k, v_{iju}^k]$ be intervals of the degree of membership and the degrees of non-membership of alternatives $x_j \in X$ on attributes $o_i \in O$, where $[\mu_{ijl}^k, \mu_{iju}^k] \subseteq [0, 1]$ and $[v_{ijl}^k, v_{iju}^k] \subseteq [0, 1]$ with $\mu_{ijl}^k + v_{ijl}^k \leq 1$. Assume that there exists a group consisting of K decision makers (or experts) e_k ($k = 1, 2, \dots, K$) denoted by $\Omega = \{e_1, e_2, \dots, e_K\}$. The group Ω has to choose one of or rank n feasible alternatives x_j ($j = 1, 2, \dots, n$) based on m attributes o_i ($i = 1, 2, \dots, m$), both quantitatively and qualitatively. Assume that $w = (w_1, w_2, \dots, w_m)^T$ is the attribute weight vector, where $0 \leq w_i \leq 1$ ($i \in M$) and $\sum_{i=1}^m w_i = 1$. In other words, decision maker e_k evaluation value of alternatives $x_j \in X$ on attributes $o_i \in O$ can be expressed as an IVIF sets $\tilde{r}_{ij}^k = ([\mu_{ijl}^k, \mu_{iju}^k][v_{ijl}^k, v_{iju}^k])$. Thus, the individual decision matrix given by DM e_k can be denoted as $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m \times n}$.

The method is mainly devoted to solving two key issues:

- (i) Determine the DM's weights with respect to different attributes;
- (ii) Obtain according to the preference relationship of the weights given by the decision maker.
- (iii) Adopt the closeness coefficient construct nonlinear programming model.

Thus, a Multi-attribute Group Decision-Making (MAGDM) problem with IVIFs can be expressed concisely in the Interval-valued matrix format and the process of resolving a MAGDM problem (see Fig.1).

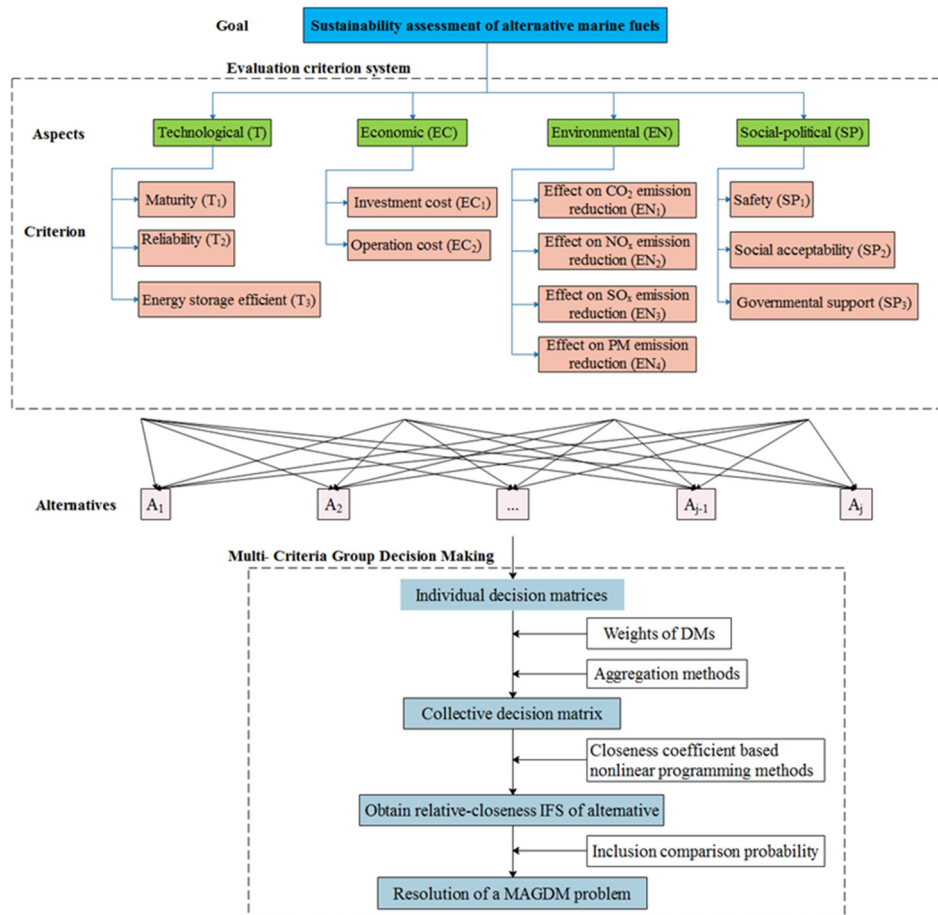


Figure 1. MAGDM apply for Evaluation of clean Alternative marine fuels

4.3. Process and Algorithm the Proposed Methodology

Step 1. Identify the evaluation alternatives $X = \{x_1, x_2, \dots, x_n\}$, attributes $O = \{o_1, o_2, \dots, o_m\}$ and decision maker $e_k (k = 1, 2, \dots, K)$. Collect the linguistic pairwise comparison matrix of criteria for each expert using Table 4 (8) and it is used to fall in Table 5.

Table 4. Linguistic scale and its corresponding IVIFs

Linguistic terms	Membership and non-membership
Absolutely Low (AL)	$([0.10,0.25],[0.65,0.75])$
Very Low (VL)	$([0.15,0.30],[0.60,0.70])$
Low(L)	$([0.20,0.35],[0.55,0.65])$
Medium Low (ML)	$([0.25,0.40],[0.50,0.60])$
Exactly Equal	$([0.50,0.50],[0.50,0.50])$
Approximately Equal (AE)	$([0.45,0.55],[0.30,0.45])$
Medium High (MH)	$([0.50,0.60],[0.25,0.40])$
High (H)	$([0.55,0.65],[0.20,0.35])$
Very High (VH)	$([0.60,0.70],[0.15,0.30])$
Absolutely High (AH)	$([0.65,0.75],[0.10,0.25])$

Table 5. Linguistic pairwise comparison matrix

kth DM i	Alternative 1	Alternative 2	...	Alternative n
Criterion 1				
Criterion 2				
...				
Criterion m				

Step 2. Convert the linguistic data in Table 5 to their corresponding Interval-valued intuitionistic fuzzy sets using Table 4 to obtain individual inter-valued intuitionistic judgement matrix \tilde{D} for each expert and the group of DMs gives the preference information Λ on the attributes' importance.(38)

Let $\tilde{D} = (\tilde{r}_{ij})_{n \times n} (\langle [\mu_{ij}^l, \mu_{ij}^u], [v_{ij}^l, v_{ij}^u] \rangle)_{m \times n}$ be an Interval-valued intuitionistic judgment matrix as follows:

$$\tilde{D} = (\langle [\mu_{ij}^l, \mu_{ij}^u], [v_{ij}^l, v_{ij}^u] \rangle)_{m \times n} = \begin{pmatrix} \langle [\mu_{11}^l, \mu_{11}^u], [v_{11}^l, v_{11}^u] \rangle & \dots & \langle [\mu_{1n}^l, \mu_{1n}^u], [v_{1n}^l, v_{1n}^u] \rangle \\ \langle [\mu_{21}^l, \mu_{21}^u], [v_{21}^l, v_{21}^u] \rangle & \ddots & \langle [\mu_{2n}^l, \mu_{2n}^u], [v_{2n}^l, v_{2n}^u] \rangle \\ \vdots & \vdots & \vdots \\ \langle [\mu_{m1}^l, \mu_{m1}^u], [v_{m1}^l, v_{m1}^u] \rangle & \dots & \langle [\mu_{mn}^l, \mu_{mn}^u], [v_{mn}^l, v_{mn}^u] \rangle \end{pmatrix}_{m \times n} \quad (11)$$

where $i(i = 1, 2, \dots, n)$ and $j(j = 1, 2, \dots, n)$ denote the criterion number.

Step 3. Calculate the weight λ_i^k of DM e_k with respect to attribute o_i ($k \in K, i \in O$) by Eq (21) (33):

(1) Calculate the similarity degree based on an extend TOPSIS

(i) Determine the positive ideal decision (PID) vector \tilde{r}_i^+ on attribute o_i .

The PID vector on attribute o_i is defined as the arithmetic average of all individual IVIF vectors \tilde{r}_i^k ($k \in K$), i.e., $\tilde{r}_i^+ = (\tilde{r}_{i1}^+, \tilde{r}_{i2}^+, \dots, \tilde{r}_{in}^+)$, where

$$\tilde{r}_{ij}^+ = ([\mu_{ijl}^*, \mu_{iju}^*], [v_{ijl}^*, v_{iju}^*]) = \text{IVIFM}_w(\tilde{r}_{ij}^1, \tilde{r}_{ij}^2, \dots, \tilde{r}_{ij}^k) \quad (j \in N) \quad (12)$$

(ii) Determine all the negative ideal decision (NID) vectors on attribute o_i .

The NID vectors on attribute o_i include the individual negative ideal decision (INID) vector, the left individual negative ideal decision (LNID) vector and the right individual negative ideal decision (RINID) vector. Denote the INID, LNID and RINID vectors on attribute o_i by $\tilde{r}_i^- = (\tilde{r}_{i1}^-, \tilde{r}_{i2}^-, \dots, \tilde{r}_{in}^-)$, $\tilde{r}_i^{l-} = (\tilde{r}_{i1}^{l-}, \tilde{r}_{i2}^{l-}, \dots, \tilde{r}_{in}^{l-})$ and $\tilde{r}_i^{r-} = (\tilde{r}_{i1}^{r-}, \tilde{r}_{i2}^{r-}, \dots, \tilde{r}_{in}^{r-})$ respectively, where

$$r_{ij}^- = ([\mu_{ijl}^-, \mu_{iju}^-], [v_{ijl}^-, v_{iju}^-]) \text{ and } \mu_{ijl}^- = v_{ijl}^*, \mu_{iju}^- = v_{iju}^*, v_{ijl}^- = \mu_{ijl}^*, v_{iju}^- = \mu_{iju}^* \quad (13)$$

$$r_{ij}^{l-} = ([\mu_{ijl}^{l-}, \mu_{iju}^{l-}], [v_{ijl}^{l-}, v_{iju}^{l-}]) \text{ and } \mu_{ijl}^{l-} = \min_k \{\mu_{ijl}^k\}, \mu_{iju}^{l-} = \min_k \{\mu_{iju}^k\}, v_{ijl}^{l-} = \max_k \{v_{ijl}^k\}, v_{iju}^{l-} = \max_k \{v_{iju}^k\}, \quad (14)$$

$$r_{ij}^{r-} = ([\mu_{ijl}^{r-}, \mu_{iju}^{r-}], [v_{ijl}^{r-}, v_{iju}^{r-}]) \text{ and } \mu_{ijl}^{r-} = \max_k \{\mu_{ijl}^k\}, \mu_{iju}^{r-} = \max_k \{\mu_{iju}^k\}, v_{ijl}^{r-} = \min_k \{v_{ijl}^k\}, v_{iju}^{r-} = \min_k \{v_{iju}^k\}. \quad (15)$$

(iii) Compute the distance $d(\tilde{r}_i^k, \tilde{r}_i^+)$, $d(\tilde{r}_i^k, \tilde{r}_i^-)$, $d(\tilde{r}_i^k, \tilde{r}_i^{l-})$ and $d(\tilde{r}_i^k, \tilde{r}_i^{r-})$ by Eq. (10)

(iv) Compute the similarity degree.

$$S_i^k = \frac{d(\tilde{r}_i^k, \tilde{r}_i^-) + d(\tilde{r}_i^k, \tilde{r}_i^{l-}) + d(\tilde{r}_i^k, \tilde{r}_i^{r-})}{d(\tilde{r}_i^k, \tilde{r}_i^+) + d(\tilde{r}_i^k, \tilde{r}_i^-) + d(\tilde{r}_i^k, \tilde{r}_i^{l-}) + d(\tilde{r}_i^k, \tilde{r}_i^{r-})} \quad (i \in m; k \in K) \quad (16)$$

where S_i^k indicated the similarity degree between vector $\tilde{r}_i^k (k \in K)$ and the PID vector \tilde{r}_i^+ , the larger the S_i^k is indicated the greater the weight λ_i^k .

(2) Calculate proximity degree using the distance measure

The proximity degree ξ_{ij}^{tk} can be compute as follow by Eq. (9)

$$\xi_{ij}^{tk} = 1 - d(\tilde{r}_{ij}^t, \tilde{r}_{ij}^k) \quad (17)$$

Compute the average proximity degree $\gamma(\tilde{r}_i^t, \tilde{r}_i^k)$ as follow

$$\gamma(\tilde{r}_i^t, \tilde{r}_i^k) = \frac{1}{n} \sum_{j=1}^m \xi_{ij}^{tk} \quad (18)$$

where represents the proximity degree between the individual information given by DM e_k and that given by DM e_t on attribute o_i .

For attribute o_i , the average proximity degree γ_i^k between DM e_k and all other DMs $e_t (t \in K, t \neq k)$ is computed as

$$\gamma_i^k = \frac{1}{s-1} \sum_{t=1, t \neq k}^K \gamma(\tilde{r}_i^t, \tilde{r}_i^k) \quad (19)$$

where the larger the γ_i^k , the bigger the weight of DM e_k on attribute o_i .

(3) Obtain the weights of DMs with respect to different attributes.

To acquire the weight of DMs between the similarity and proximity degree, Wan and Dong employ a control parameter $\delta (0 \leq \delta \leq 1)$ to construct the combined weight $\bar{\lambda}_i^k$ of DM e_k on attribute o_i as follows

$$\bar{\lambda}_i^k = \delta S_i^k + (1 - \delta) \gamma_i^k \quad (20)$$

where in practical application, we can take $\delta = 0.5$.

Normalized combined weights $\bar{\lambda}_i^k (k \in K)$, the weight $\bar{\lambda}_i^k$ of DM e_k on attribute o_i is obtained as

$$\lambda_i^k = \bar{\lambda}_i^k / \sum_{t=1}^K \bar{\lambda}_i^t \quad (i \in m; k \in K) \quad (21)$$

Step 4. Integrate all individual decision $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m \times n} (k \in K)$ into a collective IVIF matrix $\tilde{R} = (\tilde{r}_{ij})_{m \times n} = (\langle [\mu_{ij}^l, \mu_{ij}^u], [v_{ij}^l, v_{ij}^u] \rangle)_{m \times n}$ by Eq. (22).

Individual decision matrices $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m \times n}$ are integrated into collective decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$ by the IVIFWM operator Eq. (9) as follow(33):

$$\begin{aligned} \tilde{r}_{ij} &= ([\mu_{ij}^l, \mu_{ij}^u], [v_{ij}^l, v_{ij}^u]) = \sum_{k=1}^K \lambda_i^k \tilde{r}_{ij}^k \\ &= ([\sum_{k=1}^K \lambda_i^k \mu_{ijl}^k, \sum_{k=1}^K \lambda_i^k \mu_{iju}^k], [\sum_{k=1}^K \lambda_i^k v_{ijl}^k, \sum_{k=1}^K \lambda_i^k v_{iju}^k]) \end{aligned} \tag{22}$$

Thus, a MAGDM problem with IVIF sets can be concisely expressed in the Interval-valued matrix format as follows:

$$\begin{aligned} \tilde{R} &= (([\mu_{ij}^l, \mu_{ij}^u], [v_{ij}^l, v_{ij}^u]))_{m \times n} \\ &= \begin{matrix} o_1 \\ o_2 \\ \vdots \\ o_m \end{matrix} \begin{pmatrix} x_1 & \cdots & x_n \\ \langle [\mu_{11}^l, \mu_{11}^u], [v_{11}^l, v_{11}^u] \rangle & \cdots & \langle [\mu_{1n}^l, \mu_{1n}^u], [v_{1n}^l, v_{1n}^u] \rangle \\ \langle [\mu_{21}^l, \mu_{21}^u], [v_{21}^l, v_{21}^u] \rangle & \cdots & \langle [\mu_{2n}^l, \mu_{2n}^u], [v_{2n}^l, v_{2n}^u] \rangle \\ \vdots & \vdots & \vdots \\ \langle [\mu_{m1}^l, \mu_{m1}^u], [v_{m1}^l, v_{m1}^u] \rangle & \cdots & \langle [\mu_{mn}^l, \mu_{mn}^u], [v_{mn}^l, v_{mn}^u] \rangle \end{pmatrix}_{m \times n} \end{aligned} \tag{23}$$

Step 5. Pool the decision maker opinion to get a specific preference information structure on attributes, i.e., the set Λ by (40; 38).

Suppose that $w_i (i = 1, 2, \dots, m)$ are weights of the attributes $o_i \in O$, which satisfy the following normalizations: $w_i \in [0, 1] (i = 1, 2, \dots, m)$ and $\sum_{i=1}^m w_i = 1$. Let $\mathbf{w} = (w_i)_{m \times 1}$ represent a column vector of n -dimension. A set of all weight vectors is denoted by $\Lambda_0 = [\mathbf{w} = (w_i)_{m \times 1} | w_i \in [0, 1] (i = 1, 2, \dots, m), \sum_{i=1}^m w_i = 1]$. The incomplete information for the criterion weights can be generally constructed by using five basic ranking forms, which are denoted by subsets $\Lambda_s (s = 1, 2, 3, 4, 5)$ of weight vectors in Λ , respectively.

- (1) A weak ranking: $\Lambda_1 = \{w \in \Lambda_0 | w_{i_1} \geq w_{i_2}\}, i_1 \neq i_2;$
- (2) A strict ranking: $\Lambda_2 = \{w \in \Lambda_0 | 0 < a_{i_1 i_2} \leq w_{i_1} - w_{i_2} \leq b_{i_1 i_2}\}, 0 \leq a_{i_1 i_2}, b_{i_1 i_2} \leq 1, i_1 \neq i_2;$
- (3) A ranking with multiples: $\Lambda_3 = \{w \in \Lambda_0 | w_{i_1} \geq \varphi_{i_1 i_2} w_{i_2}\}, 0 \leq \varphi_{i_1 i_2} \leq 1, i_1 \neq i_2;$
- (4) A interval form: $\Lambda_4 = \{w \in \Lambda_0 | \beta_i \leq w_i \leq \beta_i + \varepsilon_i\}, 0 \leq \beta_i < \beta_i + \varepsilon_i \leq 1;$
- (5) A ranking of differences: $\Lambda_5 = \{w \in \Lambda_0 | w_{i_1} - w_{i_2} \geq w_{i_3} - w_{i_4}\}, i_1 \neq i_2 \neq i_3 \neq i_4.$

Denote by Λ the incomplete information of the attribute importance given by the DMs, which may consist of several or all of the five basic relations in Λ_0 .

Step 6. Construct auxiliary nonlinear-programming models for alternatives $x_j \in X$ using Eqs. (30) and (31)(39; 38).

In the TOPSIS, choice of reference points (the PIS and the NIS) is a sensitive problem. In order to defined the concept of the closeness coefficients, we need to determine the reference points, an IVIF positive ideal solution (IVIFPIS) and an IVIF negative ideal solution (IVIFNIS) may be defined as x^+ and x^- , respectively. IVIF sets of x^+ on attributes $o_i \in O$ may be chosen as $\{x^+, [1, 1], [0, 0]\}$, respectively. Namely, the degree of membership and the degree of non-membership of x^+ on o_i is 1 and 0, respectively. In short, x^+ denoted by $\langle [1, 1], [0, 0] \rangle$. Thus, the IVIF set vector of the IVIFPIS o_i on all attributes is expressed concisely in the vector format as $(([\mu_{il}^+, \mu_{iu}^+], [v_{il}^+, v_{iu}^+]))_{m \times 1} = (\langle [1, 1], [0, 0] \rangle)_{m \times 1}$. Similarly, the IVIF set vector of the IVIFNIS x^- on all attributes is expressed as $(([\mu_{il}^-, \mu_{iu}^-], [v_{il}^-, v_{iu}^-]))_{m \times 1} = (\langle [0, 0], [1, 1] \rangle)_{m \times 1}$.

Thus, the weighted Euclidean distances between x_j and x^+ , as well as x^- are, respectively, defined as follows:

$$d(x_j, x^+) = \sqrt{\sum_{i=1}^m \{ [w_i(1 - \mu_{ij})]^2 + (w_i v_{ij})^2 \}} \tag{24}$$

And

$$d(x_j, x^-) = \sqrt{\sum_{i=1}^m \{ [w_i \mu_{ij}]^2 + (w_i(1 - v_{ij}))^2 \}} \tag{25}$$

In a similar way to the concept of closeness coefficients in the TOPSIS, the closeness coefficient of an alternative $x_j \in X$ with respect to the IVIFPIS x^+ is defined as follows:

$$C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right) = \frac{(d(x_i, x^-))^2}{(d(x_i, x^+))^2 + (d(x_i, x^-))^2} \tag{26}$$

where $(\mu_{ij})_{m \times n}$ and $(v_{ij})_{m \times n}$ represent matrices of $m \times n$, satisfying $\mu_{ij} \in [\mu_{ij}^l, \mu_{ij}^u]$ and $v_{ij} \in [v_{ij}^l, v_{ij}^u]$; $\mathbf{w} = (w_i)_{m \times 1}$ is the weight vector in the preference information structure Λ defined as above. Obviously, $0 \leq (d(x_i, x^-))^2 \leq (d(x_i, x^-))^2 + (d(x_i, x^+))^2$.

Hence, it directly follows that:

$$0 \leq C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right) \leq 1 \tag{27}$$

According to Eqs. (24) and (25), $C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right)$ may be explicitly written out as follows:

$$C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right) = \frac{\sum_{i=1}^m \{ (w_i \mu_{ij})^2 + [w_i(1 - v_{ij})]^2 \}}{\sum_{i=1}^m \{ [w_i(1 - \mu_{ij})]^2 + (w_i v_{ij})^2 \} + \sum_{i=1}^m \{ (w_i \mu_{ij})^2 + [w_i(1 - v_{ij})]^2 \}} \tag{28}$$

where $C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right)$ is a continuous function of $m(2n+1)$ variables, including $\mu_{ij} \in [\mu_{ij}^l, \mu_{ij}^u]$ and $v_{ij} \in [v_{ij}^l, v_{ij}^u]$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) and $(w_i)_{m \times 1} \in \Lambda$. According to prove, $C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right)$ ($j = 1, 2, \dots, n$) are monotonic and non-decreasing functions of the variables $\mu_{ij} \in [\mu_{ij}^l, \mu_{ij}^u] \in (i = 1, 2, \dots, m)$ and monotonic and non-increasing functions of the variables $v_{ij} \in [v_{ij}^l, v_{ij}^u] \in (i = 1, 2, \dots, m)$.

Since $C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right) \in [0, 1]$, we can denote by $[C_j^l, C_j^u]$ derived from Eq. (28) as follows:

$$0 \leq C_j^l \leq C_j \left((\mu_{ij})_{m \times n}, (v_{ij})_{m \times n}, (w_i)_{m \times 1} \right) \leq C_j^u \leq 1 \tag{29}$$

where the μ_{ij} , v_{ij} and w_i take all values in the intervals $[\mu_{ij}^l, \mu_{ij}^u]$, $[v_{ij}^l, v_{ij}^u]$ and $[0, 1]$. Namely, the closeness coefficient of the alternative x_j to the IVIFPIS x^+ is an intuitionistic value fuzzy (IVF) set $[C_j^l, C_j^u]$ of the interval $[0, 1]$. According to the definition of the IF set, $[C_j^l, C_j^u]$ may be equivalently expressed as an IF set $C_j = \langle C_j^l, 1 - C_j^u \rangle$, which means that the closeness and non-closeness degrees of the alternative $x_j \in X$ to the IVIFPIS x^+ are C_j^l and $1 - C_j^u$, respectively.

In order to reduce the amount of calculation, solving Eqs. (33) and (34) can reduce $2mn$ variables unknown by contrast to solving Eqs. (31) and (32). Hence, they can be further simplified as follows:

$$C_j^l = \min \left\{ C_j \left((\mu_{ij}^l)_{m \times n}, (v_{ij}^u)_{m \times n}, (w_i)_{m \times 1} \right) \right. \\ \left. s. t. (w_i)_{m \times 1} \in \Lambda \right\} \tag{30}$$

and

$$C_j^u = \max \left\{ C_j \left((\mu_{ij}^u)_{m \times n}, (v_{ij}^l)_{m \times n}, (w_i)_{m \times 1} \right) \right. \\ \left. s. t. (w_i)_{m \times 1} \in \Lambda \right\} \tag{31}$$

Step 7. Solve the auxiliary nonlinear-programming models using existing nonlinear-programming methods and obtain relative-closeness IF sets $C_j = \langle C_j^l, 1 - C_j^u \rangle$ ($j = 1, 2, \dots, n$) of alternatives $x_j \in X$ to the IVIFPIS x^+ .

Step 8. Construct the inclusion comparison probability matrix $P = (p_{jh})_{n \times n}$ by pair-wise comparison of all the alternatives x_j ($j = 1, 2, \dots, n$) using Eq. (32)(38).

To make comparison between alternatives, we defined a binary relation \succcurlyeq on the set of the alternatives X . Notation “ $x_j \succcurlyeq x_h$ ” means that the alternative x_j is not worse than x_h . Let $p(x_j \succcurlyeq x_h)$ represent the probability of the event “ $x_j \succcurlyeq x_h$ ”. The probability of the IF event “ $C_j \supseteq C_h$ ” is denoted by $p(C_j \supseteq C_h)$, which is called the inclusion comparison probability of the IFSSs C_j and C_h . Hence, the inclusion comparison probability of C_j and C_h is defined as follows:

$$p(x_j \succcurlyeq x_h) = p(C_j \supseteq C_h) = \max \left\{ 1 - \max \left\{ \frac{C_h^u - C_j^l}{\pi_{C_j} + \pi_{C_h}}, 0 \right\}, 0 \right\} \tag{32}$$

where $C_j = \langle C_j^l, 1 - C_j^u \rangle$, $C_h = \langle C_h^l, 1 - C_h^u \rangle$, $\pi_{C_j} = C_j^u - C_j^l$, and $\pi_{C_h} = C_h^u - C_h^l$.

Step 9. Compute optional membership degrees θ_j of alternative x_j using Eq. (33).

In order to obtain the inclusion comparison probabilities of pair-wise alternatives in X . we can construct the matrix of possibility degree as follows

$$P = (p_{jh})_{n \times n} = \begin{matrix} & x_1 & x_2 & \dots & x_n \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{pmatrix} & & & \end{matrix}_{n \times n}$$

where $p_{jh} = p(x_j \succcurlyeq x_h) = p(C_j \supseteq C_h)$ ($j, h = 1, 2, \dots, n$) $0 \leq p_{jh} \leq 1$ and $p_{jh} + p_{hj} = 1$, which implies that P is a fuzzy complementary judgment matrix.

Then, the optimal degrees of membership for the alternatives x_j ($j = 1, 2, \dots, n$) as follows:

$$\theta_j = \frac{1}{n(n-1)} \left(\sum_{h=1}^n p_{jh} + \frac{n}{2} - 1 \right) \tag{33}$$

which the ranking order of all alternatives x_j ($j = 1, 2, \dots, n$) is generated according to the descending order of the values θ_j .

Step 10. Determine the best alternative from the set X and generate the ranking order of the alternatives x_j ($j = 1, 2, \dots, n$) according to the descending order of all optimal membership degrees θ_j .

5. A Case Study: Alternative Energy Selection in Shipping

Based on the fuel analysis in Section 3, we selected five alternative shipping fuels: LNG, Methanol, Hydrogen, Biodiesel, Ammonia. According to energy shortage and environment pollution, multiple alternative energy sources for shipping are being planned by considering the criteria and alternatives displayed in Fig.1. We presented this objective information to three experts, who have the top level of knowledge on the alternative fuel technologies and asked them to use it along with their expertise to assess the evaluation criteria as well as five alternative fuels. Objective information is helpful, but there is still a lot of knowledge for experts to bring their subjective judgments, beliefs and professional knowledge into the decision-making process.

5.1. The Application of the Proposed Methodology

The hierarchy in Fig. 1 is considered to determine the weights of criteria and sub-criteria.

Step 1 and Step 2. First, the comparison matrices for the main criteria and sub-criteria with respect to the goal are fulfilled by the three decision makers. The obtained comparison matrices are given in Table 6. The linguistic evaluations are converted to their corresponding numerical values using the scale in Table 4.

Table 6. Pairwise comparison matrices with respect to the main criteria

DM	Aspect	Alternative				
		A1	A2	A3	A4	A5
e_1	T	H	AE	MH	AE	AE
	EC	H	VH	AL	ML	VH
	EN	H	MH	VH	MH	VH
	SP	ML	EE	VH	AE	H
e_2	T	MH	MH	AH	MH	H
	EC	AH	AH	VL	MH	VH
	EN	VH	L	AH	H	AH
	SP	AE	ML	H	MH	MH
e_3	T	MH	MH	H	MH	H
	EC	VH	AH	AL	H	AH
	EN	H	AE	AH	H	VH
	SP	MH	ML	H	MH	AE

Step 3. Determine the weights of the DMs with respect to each attribute.

Calculate the weight λ_1^k of DM e_1 on attribute o_1 and the aggregated comparison matrix has been obtained from Table 6 as an example.

(i) Calculate the similarity degree of DM e_1 .

The PID vector \tilde{r}_1 for attribute o_1 is calculated by Eq.(13) as

$$\tilde{r}_1^+ = (\tilde{r}_{11}^+, \tilde{r}_{12}^+, \tilde{r}_{13}^+, \tilde{r}_{14}^+, \tilde{r}_{15}^+) = (([0.517, 0.617], [0.233, 0.383]), ([0.483, 0.583], [0.267, 0.417]), ([0.567, 0.667], [0.183, 0.333]), ([0.483, 0.583], [0.267, 0.417]), ([0.517, 0.617], [0.233, 0.383])).$$

Using Eqs. (15)-(16), INID (\tilde{r}_1^-), LNID (\tilde{r}_1^{l-}) and RNID (\tilde{r}_1^{r-}) vectors for attribute o_1 can be easily identified. Then, by Eq. (12), we have

$$d(\tilde{r}_1^1, \tilde{r}_1^+) = 0.049, d(\tilde{r}_1^1, \tilde{r}_1^-) = 0.228, d(\tilde{r}_1^1, \tilde{r}_1^{l-}) = 0.022, d(\tilde{r}_1^1, \tilde{r}_1^{r-}) = 0.087.$$

According to Eq. (17), the similarity degree of DM e_1 on attribute o_1 is obtained as $S_1^1 = 0.872$.

Table 7. Pairwise comparison matrices respect to sub-criteria

DM	Aspect	Alternative				
		A1	A2	A3	A4	A5
e_1	T1	H	ML	MH	MH	MH
	T2	EE	AE	H	AE	MH
	T3	AE	ML	MH	AE	MH
	EC1	H	VH	AL	ML	VL
	EC2	H	VL	AH	ML	AH
	EN1	H	L	AH	VH	AH
	EN2	VH	VH	MH	AH	VH
	EN3	AH	AH	AH	AH	AH
	EN4	MH	MH	VH	MH	MH
	SP1	ML	L	MH	AE	VL
	SP2	ML	MH	VH	AE	H
	SP3	H	EE	VH	AE	MH
e_2	T1	AE	AE	H	AE	MH
	T2	AE	MH	H	MH	MH
	T3	MH	ML	VH	AE	H
	EC1	VH	AH	VL	MH	L
	EC2	VH	L	VH	EE	VH
	EN1	VH	AL	AH	VH	AH
	EN2	MH	VH	VH	MH	VH
	EN3	AH	AH	AH	AH	AH
	EN4	MH	MH	H	MH	H
	SP1	AE	ML	H	MH	L
	SP2	AE	H	AH	MH	VH
	SP3	MH	ML	H	AE	MH
e_3	T1	MH	MH	H	MH	AE
	T2	ML	AE	MH	AE	EE
	T3	AE	L	H	EE	H
	EC1	AH	AH	VL	H	ML
	EC2	VH	L	AH	EE	AH
	EN1	VH	AL	AH	AH	AH
	EN2	MH	MH	AH	MH	AH
	EN3	AH	AH	AH	AH	AH
	EN4	H	H	VH	H	VH
	SP1	EE	ML	MH	AE	L
	SP2	AE	MH	H	MH	VH
	SP3	MH	AE	VH	MH	H

(ii) Calculate the proximity degree of DM e_1 .

Combining Eq. (11) with Eq. (18), we get the following proximity degree:

$$\xi_{11}^{21}=0.950, \xi_{12}^{21}=0.950, \xi_{13}^{21}=0.850, \xi_{14}^{21}=0.950, \xi_{15}^{21}=0.900.$$

According to Eq. (19), the average proximity degree between \tilde{r}_1^2 and \tilde{r}_1^1 is calculated as

$$\gamma(\tilde{r}_1^2, \tilde{r}_1^1) = \frac{1}{4} \sum_{j=1}^5 \xi_{1j}^{21} = 0.9200.$$

Likewise, we obtain $\gamma(\tilde{r}_1^3, \tilde{r}_1^1) = \frac{1}{4} \sum_{j=1}^5 \xi_{1j}^{21} = 0.9400$.

By using Eq. (20), the average proximity degree between DM e_1 and the other three DMs is computed as

$$\gamma_1^1 = \frac{1}{2} \sum_{t=2}^3 \gamma(\tilde{r}_1^t, \tilde{r}_1^1) = 0.9300.$$

(iii) Determine the weight of DM e_1 on attribute o_1 .

Taking $\delta = 0.5$ in Eq. (21), the combined weight of DM e_1 on attribute o_1 is obtained as $\bar{\lambda}_1^1 = 0.9010$.

By normalizing $\bar{\lambda}_1^1, \bar{\lambda}_1^2$ and $\bar{\lambda}_1^3$, the weights of the three DMs with respect to o_1 are obtained as follows:

$$\lambda_1^1 = 0.3334, \lambda_1^2 = 0.3236, \lambda_1^3 = 0.3427.$$

In the same way, the DM's weights on the other attribute can be compared and are shown in [Table 8](#).

Table 8. Weights of each DM with respect to different attributes in main criteria

	T	EC	EN	SP
e_1	0.3239	0.3236	0.3328	0.3190
e_2	0.3334	0.3396	0.3246	0.3446
e_3	0.3427	0.3368	0.3426	0.3363

Table 9. Weight of each DM with respect to different attribute in sub-criteria

	T1	T2	T3	EC1	EC2	EN1	EN2	EN3	EN4	SPI	SP2	SP3
e_1	0.3253	0.3414	0.3278	0.3222	0.3258	0.3290	0.3261	0.3172	0.3341	0.3305	0.3257	0.3359
e_2	0.3392	0.3372	0.3289	0.3421	0.3355	0.3365	0.3414	0.3656	0.3368	0.3329	0.3363	0.3303
e_3	0.3355	0.3214	0.3433	0.3357	0.3387	0.3346	0.3324	0.3172	0.3291	0.3366	0.3380	0.3339

In [Table 7](#), the pairwise comparison matrices for the sub-criteria with respect to the main criteria are presented. The same calculation procedure as the main criteria is applied for the pairwise comparison matrices of the sub-criteria. The DM's weights on the other attribute can be compared about sub-criteria and are shown in [Table 9](#). The local and global weights of the sub-criteria are given in [Table 10](#).

Step 4. Obtain the collective matrix.

By using Eq. (23), the collective IVIF matrix is acquired as follows:

Table 10. Local and global weights of sub-criteria

	e_1	e_2	e_3		e_1	e_2	e_3	λ_i^1	λ_i^2	λ_i^3
T	0.901	0.928	0.953	T1	0.887	0.925	0.915	0.316	0.339	0.345
				T2	0.902	0.891	0.850	0.332	0.338	0.331
				T3	0.864	0.867	0.905	0.318	0.320	0.353
EC	0.893	0.937	0.929	EC1	0.888	0.943	0.925	0.313	0.348	0.339
				EC2	0.914	0.941	0.950	0.316	0.342	0.342
EN	0.921	0.899	0.948	EN1	0.952	0.974	0.969	0.329	0.328	0.344
				EN2	0.901	0.943	0.918	0.326	0.333	0.342
				EN3	0.868	1.000	0.868	0.317	0.357	0.326
				EN4	0.949	0.957	0.935	0.334	0.328	0.338
SP	0.860	0.929	0.906	SP1	0.901	0.907	0.917	0.316	0.344	0.340
				SP2	0.902	0.931	0.936	0.312	0.348	0.341
				SP3	0.917	0.902	0.911	0.322	0.342	0.337

$$\tilde{R} = \left(\begin{array}{l} ([0.499,0.599], [0.251,0.401])([0.404,0.520], [0.346,0.480])([0.534,0.634], \\ ([0.400,0.484], [0.433,0.516])([0.467,0.567], [0.283,0.433])([0.533,0.633], \\ ([0.466,0.566], [0.284,0.434])([0.232,0.382], [0.518,0.618])([0.551,0.651], \\ ([0.601,0.701], [0.149,0.299])([0.634,0.734], [0.116,0.266])([0.134,0.284], \\ ([0.584,0.684], [0.166,0.316])([0.184,0.334], [0.566,0.666])([0.633,0.733], \\ ([0.584,0.684], [0.166,0.316])([0.133,0.283], [0.617,0.717])([0.650,0.750], \\ ([0.533,0.633], [0.217,0.367])([0.566,0.666], [0.184,0.334])([0.585,0.685], \\ ([0.650,0.750], [0.100,0.250])([0.650,0.750], [0.100,0.250])([0.650,0.750], \\ ([0.517,0.617], [0.233,0.383])([0.517,0.617], [0.233,0.383])([0.584,0.684], \\ ([0.404,0.486], [0.431,0.514])([0.234,0.384], [0.516,0.616])([0.517,0.617], \\ ([0.388,0.503], [0.362,0.497])([0.517,0.617], [0.233,0.383])([0.600,0.700], \\ ([0.516,0.616], [0.234,0.384])([0.398,0.483], [0.433,0.517])([0.583,0.683], \\ [0.216,0.366])([0.483,0.583], [0.267,0.417])([0.483,0.583], [0.267,0.417]) \\ [0.217,0.367])([0.467,0.567], [0.283,0.433])([0.500,0.567], [0.333,0.433]) \\ [0.217,0.367])([0.468,0.532], [0.371,0.468])([0.534,0.634], [0.216,0.366]) \\ [0.217,0.367])([0.439,0.554], [0.311,0.446])([0.167,0.317], [0.583,0.683]) \\ [0.117,0.267])([0.421,0.468], [0.500,0.532])([0.633,0.733], [0.117,0.267]) \\ [0.100,0.250])([0.617,0.717], [0.133,0.283])([0.650,0.750], [0.100,0.250]) \\ [0.165,0.315])([0.549,0.649], [0.201,0.351])([0.617,0.717], [0.133,0.283]) \\ [0.100,0.250])([0.650,0.750], [0.100,0.250])([0.650,0.750], [0.100,0.250]) \\ [0.166,0.316])([0.517,0.617], [0.233,0.383])([0.550,0.650], [0.200,0.350]) \\ [0.233,0.383])([0.467,0.567], [0.283,0.433])([0.184,0.334], [0.566,0.666]) \\ [0.150,0.300])([0.484,0.584], [0.266,0.416])([0.584,0.684], [0.166,0.316]) \\ [0.167,0.317])([0.467,0.567], [0.283,0.433])([0.517,0.617], [0.233,0.383]) \end{array} \right)$$

Step 5. According to the experience, knowledge and preference, the decision maker may give a specific preference information structure Λ on the attribute o_i ($i = 1, 2, \dots, 12$), where the Λ is given as follows:

$$\Lambda = \{\omega \in \Lambda_0 | 0.05 \leq \omega_1 \leq 0.08, 0.03 \leq \omega_2 \leq 0.06, 0.02 \leq \omega_3 \leq 0.05, \\ 0.08 \leq \omega_4 \leq 0.12, 0.12 \leq \omega_5 \leq 0.15, 0.15 \leq \omega_6 \leq 0.2, \\ 0.08 \leq \omega_7 \leq 0.1, 0.12 \leq \omega_8 \leq 0.15, 0.05 \leq \omega_9 \leq 0.1, \\ 0.05 \leq \omega_{10} \leq 0.08, 0.01 \leq \omega_{11} \leq 0.03, 0.02 \leq \omega_{12} \leq 0.04,$$

$$\begin{aligned} 1.5\omega_2 \leq \omega_1, 3\omega_3 \leq \omega_1, 1.5\omega_4 \leq \omega_5, \omega_8 \leq \omega_6, \omega_7 \leq \omega_8, \\ 2.8\omega_{11} \leq \omega_{10}, \omega_2 + \omega_3 \leq \omega_1\omega_6 \leq \omega_7 + \omega_8 + \omega_9, \\ \omega_{11} + \omega_{12} \leq \omega_{10} \} \end{aligned}$$

Step 6. According to Eqs. (33) and (34), two auxiliary nonlinear programming models can be constructed for the alternative x_i as follow

$$\begin{aligned} C_1^l = \min\{ & (0.499\omega_1^2 + 0.400\omega_2^2 + 0.466\omega_3^2 + 0.601\omega_4^2 + 0.584\omega_5^2 + 0.584\omega_6^2 \\ & + 0.533\omega_7^2 + 0.650\omega_8^2 + 0.517\omega_9^2 + 0.404\omega_{10}^2 + 0.388\omega_{11}^2 + 0.516\omega_{12}^2 \\ & + 0.599\omega_1^2 + 0.484\omega_2^2 + 0.566\omega_3^2 + 0.701\omega_4^2 + 0.684\omega_5^2 + 0.684\omega_6^2 \\ & + 0.633\omega_7^2 + 0.750\omega_8^2 + 0.617\omega_9^2 + 0.486\omega_{10}^2 + 0.503\omega_{11}^2 + 0.616\omega_{12}^2) \\ & / (0.499\omega_1^2 + 0.400\omega_2^2 + 0.466\omega_3^2 + 0.601\omega_4^2 + 0.584\omega_5^2 + 0.584\omega_6^2 \\ & + 0.533\omega_7^2 + 0.650\omega_8^2 + 0.517\omega_9^2 + 0.404\omega_{10}^2 + 0.388\omega_{11}^2 + 0.516\omega_{12}^2 \\ & + 0.599\omega_1^2 + 0.484\omega_2^2 + 0.566\omega_3^2 + 0.701\omega_4^2 + 0.684\omega_5^2 + 0.684\omega_6^2 \\ & + 0.633\omega_7^2 + 0.750\omega_8^2 + 0.617\omega_9^2 + 0.486\omega_{10}^2 + 0.503\omega_{11}^2 + 0.616\omega_{12}^2 \\ & + 0.501\omega_1^2 + 0.600\omega_2^2 + 0.534\omega_3^2 + 0.399\omega_4^2 + 0.416\omega_5^2 + 0.416\omega_6^2 \\ & + 0.467\omega_7^2 + 0.350\omega_8^2 + 0.483\omega_9^2 + 0.596\omega_{10}^2 + 0.612\omega_{11}^2 + 0.484\omega_{12}^2 \\ & + 0.401\omega_1^2 + 0.516\omega_2^2 + 0.434\omega_3^2 + 0.299\omega_4^2 + 0.316\omega_5^2 + 0.316\omega_6^2 \\ & + 0.367\omega_7^2 + 0.250\omega_8^2 + 0.383\omega_9^2 + 0.514\omega_{10}^2 + 0.497\omega_{11}^2 + 0.384\omega_{12}^2) \} \end{aligned}$$

$$s. t. (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}, \omega_{11}, \omega_{12})^T \in \Lambda$$

and

$$\begin{aligned} C_1^u = \max\{ & (0.599\omega_1^2 + 0.484\omega_2^2 + 0.566\omega_3^2 + 0.701\omega_4^2 + 0.684\omega_5^2 + 0.684\omega_6^2 \\ & + 0.633\omega_7^2 + 0.750\omega_8^2 + 0.617\omega_9^2 + 0.486\omega_{10}^2 + 0.503\omega_{11}^2 + 0.616\omega_{12}^2 \\ & + 0.749\omega_1^2 + 0.567\omega_2^2 + 0.716\omega_3^2 + 0.851\omega_4^2 + 0.834\omega_5^2 + 0.834\omega_6^2 \\ & + 0.783\omega_7^2 + 0.900\omega_8^2 + 0.767\omega_9^2 + 0.569\omega_{10}^2 + 0.638\omega_{11}^2 + 0.766\omega_{12}^2) \\ & / (0.599\omega_1^2 + 0.484\omega_2^2 + 0.566\omega_3^2 + 0.701\omega_4^2 + 0.684\omega_5^2 + 0.684\omega_6^2 \\ & + 0.633\omega_7^2 + 0.750\omega_8^2 + 0.617\omega_9^2 + 0.486\omega_{10}^2 + 0.503\omega_{11}^2 + 0.616\omega_{12}^2 \\ & + 0.749\omega_1^2 + 0.567\omega_2^2 + 0.716\omega_3^2 + 0.851\omega_4^2 + 0.834\omega_5^2 + 0.834\omega_6^2 \\ & + 0.783\omega_7^2 + 0.900\omega_8^2 + 0.767\omega_9^2 + 0.569\omega_{10}^2 + 0.638\omega_{11}^2 + 0.766\omega_{12}^2 \\ & + 0.401\omega_1^2 + 0.516\omega_2^2 + 0.434\omega_3^2 + 0.299\omega_4^2 + 0.316\omega_5^2 + 0.316\omega_6^2 \\ & + 0.367\omega_7^2 + 0.250\omega_8^2 + 0.383\omega_9^2 + 0.514\omega_{10}^2 + 0.497\omega_{11}^2 + 0.384\omega_{12}^2 \\ & + 0.251\omega_1^2 + 0.433\omega_2^2 + 0.284\omega_3^2 + 0.149\omega_4^2 + 0.166\omega_5^2 + 0.166\omega_6^2 \\ & + 0.217\omega_7^2 + 0.100\omega_8^2 + 0.233\omega_9^2 + 0.431\omega_{10}^2 + 0.362\omega_{11}^2 + 0.234\omega_{12}^2) \} \end{aligned}$$

$$s. t. (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}, \omega_{11}, \omega_{12})^T \in \Lambda$$

Step 7. Using existing nonlinear programming methods, optimal objective function values of the above two nonlinear programming model are obtained as follows:

$$C_1^l = 0.696, C_1^u = 0.897,$$

Respectively, the closeness IF set of the alternative x_1 is $C_1 = \langle 0.696, 0.103 \rangle$.

In the same way, the closeness IF sets of the alternatives x_j ($j = 2, 3, 4, 5$) are obtained as follows:

$$C_2 = \langle 0.209, 0.349 \rangle, C_3 = \langle 0.703, 0.098 \rangle, C_4 = \langle 0.627, 0.146 \rangle, C_5 = \langle 0.665, 0.895 \rangle$$

respectively.

Step 8 and Step 9. Using Eq. (35), the inclusion probability of the IF sets C_1 and C_3 can be calculated as follows:

$$p(x_1 \supseteq x_3) = p(C_1 \supseteq C_3) = \max\left\{1 - \max\left\{\frac{0.902-0.696}{(0.897-0.696)+(0.902-0.703)}, 0\right\}, 0\right\}=0.486$$

According to $p_{jh} + p_{hj} = 1$, it easily follows that $p(C_1 \subseteq C_3) = 1 - 0.486 = 0.514$.

Likewise, the inclusion comparison probabilities of all other IF sets can be obtained and expressed in the matrix format as follows:

$$P = \begin{pmatrix} 0.500 & 1.000 & 0.354 & 0.482 & 0.430 \\ 0.000 & 0.500 & 0.000 & 0.000 & 0.000 \\ 0.646 & 1.000 & 0.500 & 0.624 & 0.569 \\ 0.518 & 1.000 & 0.376 & 0.500 & 0.449 \\ 0.570 & 1.000 & 0.431 & 0.551 & 0.500 \end{pmatrix}$$

Using Eq. (36) with P matrix and $n=5$, the optimal membership degrees of the alternatives x_1 are computed as follows:

$$\begin{aligned} \theta_1 &= \frac{1}{n(n-1)} \left(\sum_{h=1}^n p_{1h} + \frac{n}{2} - 1 \right) \\ &= \frac{1}{5(5-1)} \left[(0.500 + 1.000 + 0.354 + 0.482 + 0.430) + \frac{5}{2} - 1 \right] = 0.233 \end{aligned}$$

In the same way, the optimal membership degrees of the alternatives x_j ($j = 2,3,4,5$) are obtained as follows:

$$\theta_2 = 0.100, \theta_3 = 0.242, \theta_4 = 0.217, \theta_5 = 0.228.$$

Step 10. It is easy that to see that the decreasing order of the optimal membership degrees of the alternatives x_j ($j = 1,2,3,4,5$) is obtained as follows:

$$\theta_3 > \theta_5 > \theta_4 > \theta_1 > \theta_2$$

5.2. Sensitivity Analysis

Since the attribute weights used in this article are incomplete information, in order to observe the influence of a possible change in the weight of an attribute on the decision of alternative fuel selection, sensitivity analysis is carried out in the following. We selected two attributes, such as operating cost (EC₂) and effect on CO₂ emission reduction efficiency(EN₁), and changed the incomplete information of attribute preferences to observe whether it will affect the choice of alternative fuel.

In Fig. 2, collect the opinions of decision makers to obtain a specific preference information structure about the attribute EC₂, $\Lambda_2 = \{w \in \Lambda_0 | 0.12 \leq w_5 \leq 0.15\}$. When the preference structure of operation cost is changed, the ranking of the five alternative fuels changes accordingly. When $w_5=0.16$ or 0.11 , LNG and hydrogen fuel rank the same, when $w_5=0.17$, LNG fuel becomes the best alternative fuel.

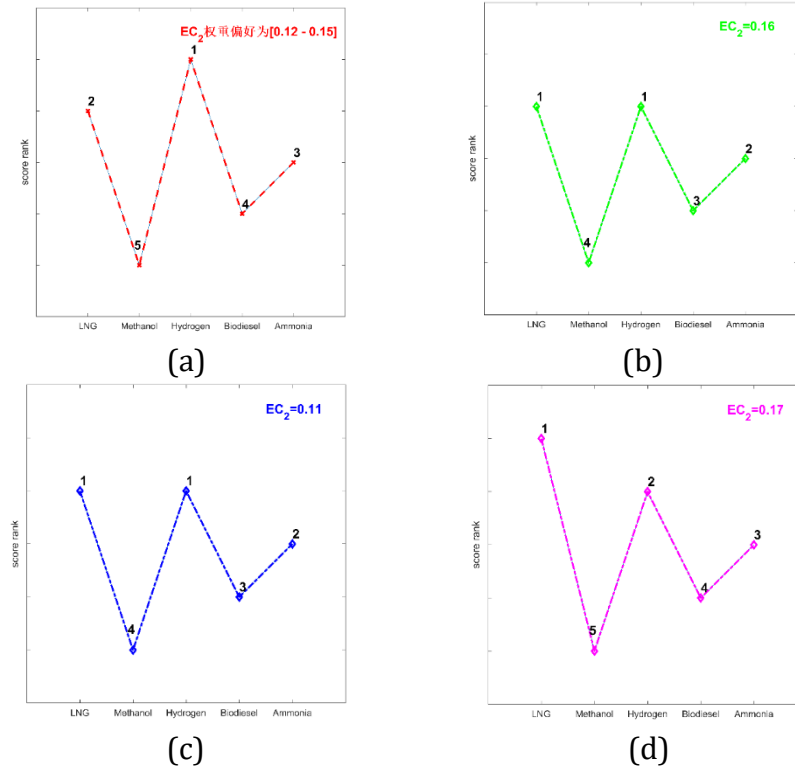
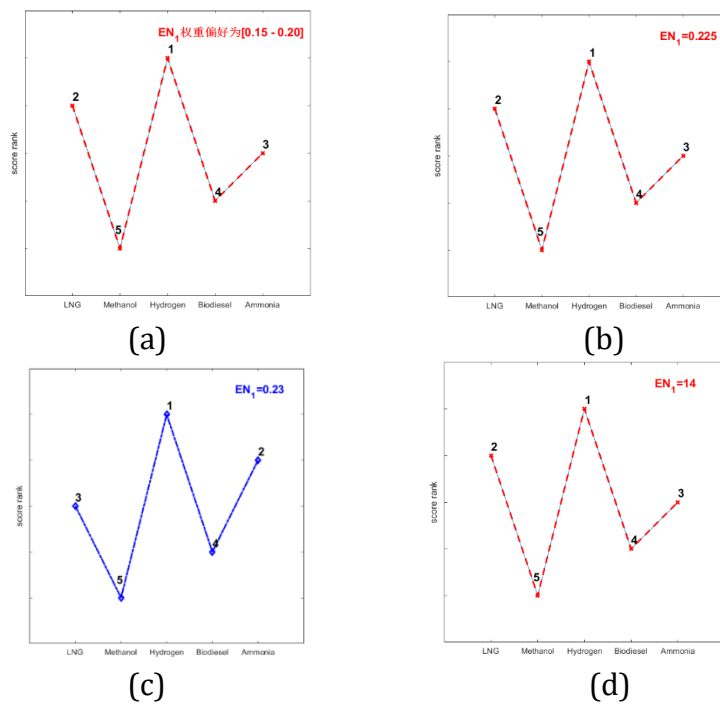


Figure 2. Preference information structure about the operation cost

In Fig. 3, collect the opinions of decision makers to obtain a specific preference information structure about the attribute EN_1 , $\Lambda_2 = \{w \in \Lambda_0 | 0.152 \leq w_6 \leq 0.20\}$. When the preference structure of operation cost is changed, the ranking of the five alternative fuels changes accordingly. When $w_6=0.225$ or 0.14, changes in the attribute preference structure did not change the ranking of alternative fuels. When $w_6=0.23$, Hydrogen is still the best alternative fuel but ammonia is superior to LAN fuel, ranking second. When $w_6=0.135$, LNG and hydrogen fuel rank the same. When $w_6=0.13$, LNG fuel becomes the best alternative fuel.



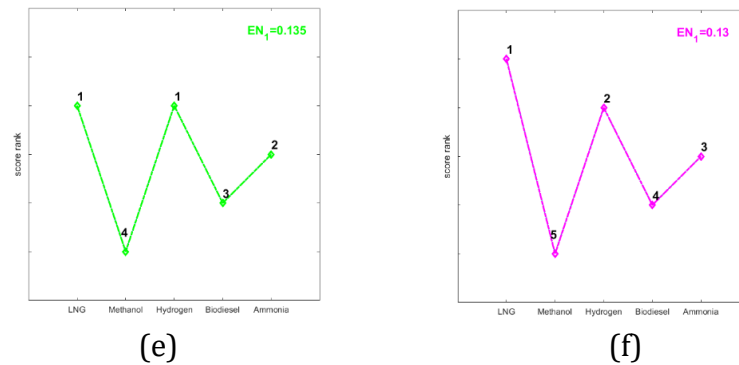


Figure 3. Preference information structure about the effect on CO₂ emission reduce

The preference structure of the attributes in this paper is partially known, among which, changing the preference structure of a certain attribute may not change the ranking of alternative fuels for shipping.

6. Conclusion

This research focuses on the decision-making of alternative fuels for ships, which requires the establishment of a large number of evaluation standard frameworks. Aiming at the problem of choosing clean energy for ships, a method based on the combination of Interval-valued intuitionistic fuzzy sets and the concept of closeness coefficient similar to TOPSIS is applied. This method uses a Multi-attribute group decision-making method to determine the weights of experts in different fields. The IVIF set represents the degree of membership/satisfaction and the degree of non-membership/dissatisfaction of alternatives on attribute, and the preference information of the attribute is incomplete, thus establishing an auxiliary nonlinear programming model. Sensitivity analysis, changing the preference structure of a certain attribute, the ranking of alternative fuels may change. This means that the decision-making method we adopt is effective for changing attribute preferences. This method can express the uncertainty of language and the preference of information in the decision-making process, and define the membership function in detail, so as to better express the way of thinking of human beings.

In this study, five alternative ship fuel options were selected for evaluation, and twelve evaluation criteria were given. However, there are a large number of literatures that have studied alternative fuels for ships, and there are countless criteria to consider, which increases the difficulty of establishing standard evaluation indicators. In the future, we hope to contribute to the shipping industry by studying a variety of clean fuels. In addition, the evaluation criterion selected in this article are not perfect. It is recommended to establish a set of evaluation criterion with wide applicability when further evaluating alternative fuels for ships. Finally, other extensive MCDM methods, such as VIKOR and ELECREE, can be used to further extend the same problem using fuzzy sets, and the results obtained are compared with this article.

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