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# Study on the Vehicle Routing Problem with Selective Pickup and Delivery and Multiple Visits to Pickup Nodes

Xuelei Wang

School of management, Shanghai University, Shanghai 200444, China

## Abstract

**Based on the actual logistics distribution scenario in omni-channel retailing, this paper proposes a new variant of VRPPD. The new problem considers the characteristics of time window, the selection of pickup nodes and multiple visits to pickup nodes. Based on the classical data set in related research, a set of new data set of new model are generated. In this paper, CPLEX is used to solve the new problem. Comparing with the vehicle routing problem with one pickup node and delivery and single visit. The complexity and efficiency of the new model are verified. The results show that the new model can reduce the distribution cost of the distribution network and the number of vehicles used. The research can provide advice for the optimization of distribution newtwork in omni-channel retailing.**

## Keywords

**Vehicle Routing Problem with Pickup and Delivery; Selective Pickup Node; Pickup with Split Loads; Multi-visit for One Pickup Node.**

## 1. Introduction

With the development of Internet and e-commerce, the omni-channel retailing has become the trend of retailing, which centering on consumer experience and integrating multiple online and offline retail channels [1]. The channel coordination between online e-commerce and offline retail stores enables logistics personnel to pick up goods from e-commerce warehouses or offline retail stores close to consumers to meet the commodity delivery demands of terminal consumer, thus one delivery node may correspond to multiple selective pickup nodes when meeting a pair of pickup and delivery demands. In addition, because the pickup is arranged according to the delivery demands of consumers, sometimes the time window constraints of all consumers are too tight, or the loading of vehicles is limited, a pickup node can be visited by the same vehicle or different vehicles for many times.

With the intensification of market competition and the continuous upgrading of consumption, consumers have higher demand for the timeliness and service quality of logistics distribution in the last mile. At present, logistics distribution still has the problems of high cost and untimely distribution, which directly affects the shopping experience of consumers, is not conducive to the development of omni-channel retailing, and is not conducive to the development of retail enterprises and logistics enterprises. Therefore, how to optimize the terminal distribution network under the omni channel retailing, reduce the logistics distribution cost and improve the consumer user experience is a very important and necessary problem faced by both retail enterprises and logistics enterprises.

The core of logistics distribution optimization is to optimize the route of distribution vehicles. Reasonably arranging the driving route of vehicles can provide customers with satisfactory logistics services, save logistics distribution costs and increase the income of retail enterprises or logistics enterprises. In the existing vehicle routing problem with pickup and delivery (VRPPD), pickup node and delivery node are paired. And a pickup node can only be visited by one vehicle once. The existing VRPPD rarely considers the selection of pickup nodes, and rarely involves a pickup node is visited visited multiple times by one or more vehicles, which is not applicable to the logistics distribution scenario under the omni channel retailing. Therefore, according to the characteristics of logistics distribution under the omni-channel retailing, this paper introduces the characteristics of pickup node selection, time window, pickup with split loads, Multi-visit for one pickup node into the existing VRPPD, taking the vehicle running cost as the objective, the vehicle routing problem with selective pickup and delivery and multiple visits to pickup nodes(VRPSPDMV) is proposed.

The contributions of this paper are as follows:

(1) This study formulates a new model of VRPPD. The new model considers the selective pickup node for one delivery node, time windows, split pickup with split loads and Multi-visit for one pickup node. The new model breaks the assumption that delivery nodes and pickup nodes are paired, making the relationship between supply and demand become a decision variable that can be optimized. The problem also relaxes the constraint of one pickup node only be visited by one vehicle once a time, llowing the vehicle can be visited by the same vehicle or different vehicles. So the vehicle can flexibly pick up goods according to the demand of the delivery nodes.

(2) CPLEX is used to solve the VRPSPDMV, and the effectiveness of the VRPSPDMV is verified by comparing the model proposed in this paper with the similar VRPPD model.

The remainder of this paper is organized as follows. Section 2 reviews related work on the VRPPD. Section 3 presents the formal formulation of the VRPSPDMV. The experimental results are presented and discussed in Section 4. Finally, conclusions and recommend the directions for future work are given in Section 5.

## 2. Related Work

VRPPD is also known as pickup and delivery problem(PDP). Berbeglia et al. [2] conducted a comprehensive survey of VRPPD formulations and classified them into many-to-many(M-M) schemes, one-to-one(1-1) structure and one-to-many-to-one(1-M-1) structure. The (M-M) structure assumes that the vehicle collects commodities from many pickup nodes and supplies them to many delivery nodes. In the (1-1) structure, paired pickup and delivery nodes are ordinarily considered, and each commodity has its designate source and destination. The (1-M-1) structure involves two types of commodities that originate from and terminate at the depot respectively. In the real world, sometimes vehicles don't need to visit all the pickup nodes, for some purposes, just visiting a few pickup nodes can satisfy the demand. Some papers have studied the VRPPD of considering the selection of pickup node.

Some papers have studied pickup node selection under the (M-M) structure. This kind of study is to minimize the cost as the goal in general. The VRP with selective pickup nodes and delivery with a single vehicle(VRPSPD) are considered in some papers(Ho and Szeto[3], Ting and Liao [4], Rafael [5], Mou and Dai [6]. ). The objective of these papers are to minimize the total travel cost of the vehicle. Liao and Ting [7] aimed to minimize travel distance and vehicle capacity required. Ting et al. [8] considered VRPSP with multiple vehicles and. Above all focus on applications that bicycle rebalancing problem(BRP) or wireless sensor network(WSN). They didn’t consider time windows constrains. Takada et al. [9] focused on a subproblem of VRPSPD with time windows and Multi-visit for pickup nodes, this problem is called the optimal pickup pointproblem(OPP).

Some papers have studied pickup node selection under the (1-1) structure. A request includes a pickup point and a delivery point and a request is associated with revenue, and a revenue can be obtained if arequest is met. In the case of constraints on route duration, the request are selective according to the maximum total revenue. Therefore,under the (1-1) structure, the goal is usually to consider the minimum cost and the maximum revenue. Gansterer et al. [10], Alhujaylan and Hosny [11] studied the multi-vehicle profitable pickup and delivery problem(MVPPDP). Chami et al. [12] studies the bi-objective selective pickup and delivery problem with time windows and paired demands(SPDPTWPD). Peng et al. [13] considered SPDPTWPD with transfers. Riedler and Raidl [14]consider both the goal of maximum benefit and user inconvenience in the Dial-A-Ride Problem(DARP). Sun et al. [15] studied the time-dependent SPDPTWPD.

Some papers have studied pickup node selection under the (1-M-1) structure. This kind of study is to minimize the difference between cost and revenue. Gribkovskaia et al. [16] studied the situation in which vehicles selectively pick up goods from some nodes while meeting the delivery demand, so as to obtain some additional income. They call it single vehicle routing problem with selective pickups (SVRPDSP). The objective function of this study is to minimize the net cost of the total vehicle cost minus the additional income. Gabriel et al. [17] added time windows constraints to SVRPDSP. Assis et al. [18] considered traveling cost and the number of pickup nodes which are not visited of SVRPDSP. Bruck and Santos [19] studied the multiple of SVRPDSP.

In the above studies, few studies took into account the characteristics of time window, selective pickup nodes, pickup with split loads, Multi-visit for one pickup node. The above models are not suitable for omni-channel retail distribution scenarios. Therefore, a new variant of VRPPD is proposed in this paper, which takes into account these characteristics. In the omni-channel retail distribution scenario, a delivery node can accept goods from multiple pickup nodes, and a pickup node can supply to multiple deliver nodes, so the model in this paper belongs to the (M-M) structure. In the following, we address the VRPSPDMV.

# 3. The Vehicle Routing Problem with Selective Pickup and Delivery and Multiple Visits to Pickup Nodes

## 3.1. Problem Seting

The distribution scenario constructed in this paper is shown in Figure 1. It is assumed that there are multiple supply stores in a certain area, representing multiple pickup nodes. After customers place orders for different goods, a certain number of door-to-door delivery orders are generated. The locations of these customers represent delivery nodes. The number of pick-up nodes is less than the number of delivery nodes, so some pickup nodes can supply goods to multiple delivery nodes. Due to the delivery node demand goods are different, some delivery node demand goods only have inventory in one pickup node, and some delivery nodes have inventory in multiple pickup nodes, for the delivery node with multiple pickup nodes supply, any one of these pickup nodes can meet the supply demand. Now there are a certain number of vehicles in the depot. After starting from the depot, the vehicle selects the appropriate pickup node to pick up the goods, and then delivers the goods to the corresponding delivery node, and finally returns to the depot. Each pickup node can be visited multiple times by the same or different vehicles. Each delivery node can only be visited once. In the case of satisfying the requirements of all delivery nodes, the sum of the travel cost of all vehicle are minimized.

Assume that there are $m$ pickup nodes and $n$ delivery nodes in the distribution area, 0 represents depot, the demand of depot is 0. $P=\{1,2,...,m\}$ is the set of pickup nodes and $D=\{1,2,...,n\}$ is the set of delivery nodes. The goods required by each delivery node can be supplied by at least one pickup node. Some pickup nodes supply goods to only one delivery node, while others supply goods to multiple delivery nodes. $K=\{1,2,...,K_{num}\}$ is the set of vehicles belongs to depot, the maximum loading capacity of distribution vehicles is $Q$, and each node has time window constraints. The needs of each delivery node are met by at least one specific pickup node. Some pickup nodes can meet to the demands of multiple delivery nodes.

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Since the model proposed in this paper allows pickup one pickup nodes to be visited by the same or different vehicles for multiple times, in order to facilitate multiple visits for one pickup node, the way Luan et al. [20] is used for reference to split pickup nodes that can provide goods to multiple delivery nodes. In order to avoid confusion, new notations is used to represent the dummy nodes be split up from the original pickup node. Figure2 shows an example of spliting one pickup node. The rules for splitting nodes are as follows:

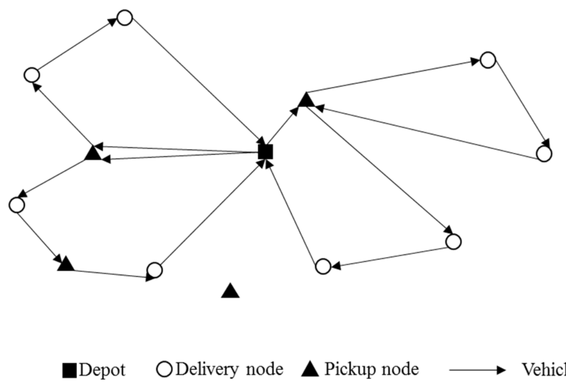


**Figure 1.** Logistics distribution scenario of VRPSPMV

(1) Assuming that one pickup node in the distribution network can provide goods to $l$ delivery nodes, the pickup point is split up $l$ dummy pickup nodes. Each dummy pickup node only supplies goods to the corresponding delivery node, and the supply quantity of each dummy pickup node is equal to the demand of the corresponding delivery node. The total supply of dummy pickup nodes split from the same pickup node is equal to the supply of the original unsplit pickup node.

(2) For the dummy pickup nodes split from one pickup node, the location and time window of each dummy pickup node are the same as the original pickup node. To simplify the problem, the service time of each dummy pickup node is the average time from the original pickup node to each delivery node served.

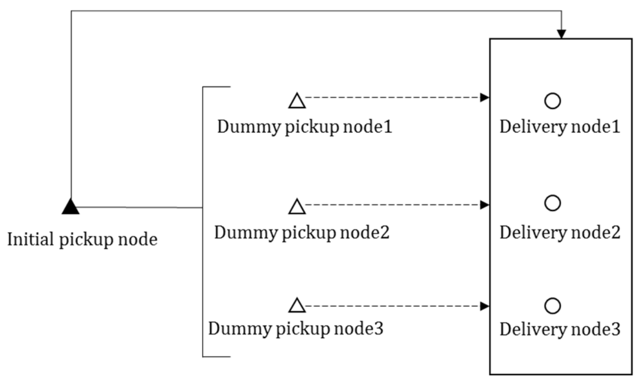


**Figure 2.** One splitting pickup node

The following mathematical model is constructed according to the distribution network after splitting the pickup nodes.

## 3.2. Problem Formulation

The notations of VRPSPDMV are given below:

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**Table 1.** The notations of VRPSPDMV

| | |
|---|---|
| $K_{num}$ | the initial number of vehicle given |
| $0$ | depot |
| $n$ | the number of delivery nodes |
| $m^u$ | the number of split pickup nodes, $m^u > n$ |
| $i, j, h, p, d$ | id of node |
| $nD_i^u$ | The ith delivery node corresponds to the total number of available pickup nodes |
| $nP_i^u$ | The ith pickup node corresponds to the total number of available delivery nodes |
| $k$ | id of vehicle |
| $c_{ij}^u$ | the cost per unit distance from node i to node j, $c_{ij}^u$=1 |
| $d_{ij}^u$ | the Euclidean distance from $i$ to $j$ |
| $t_{ij}^u$ | the traveling time from $i$ to $j$ |
| $s_i^u$ | the loading or unloading duration of the node $i$ |
| $[e_i^u, l_i^u]$ | the earliest and latest start time for loading and unloading of node $i$ |
| $q_i^u$ | the demand of node $i$, $q_i^u$>0 means that the goods are loaded at this node, $q_i^u$<0 means that the goods are unloaded at this node, and the demand for depot is 0 |
| $Q$ | the max capacity of the vehicle |
| $M$ | an arbitrarily large positive integer |
| $D$ | set of delivery nodes, $D$:={1,2,...,$n$} |
| $P^u$ | set of pickup nodes, $P^u := \{n+1, n+2, ..., n+m^u\}$ |
| $P_i^u$ | set of pickup nodes that any of them can provide goods to delivery node $i$, and elements in the set are known in advance |
| $D_i$ | set of delivery nodes that pickup point $i$ can supply goods, and elements in the set are known in advance |
| $Nodes^u$ | set of all nodes except depot, $Nodes := \{P^u \cup D\}$ |
| $N^u$ | all nodes, $N := \{0 \cup P^u \cup D\}$ |
| $K$ | set of vehicles, $K := \{1, 2, ..., K_{num}\}$ |
| $x_{kij}^u$ | is 1 when the vehicle $k$ travels from node $i$ to node $j$, otherwise is 0 |
| $z_{kij}^u$ | the delivery node $i$ is supplied by the pickup node $j \in P_i^u$ and the request is servered by vehicle $k$ is 1, otherwise 0 |
| $S_i^{uk}$ | time of vehicle $k$ when vehicle k actually visits node $i$, $S_0^{uk}$=0 |
| $Q_i^{uk}$ | capacity of vehicle $k$ when vehicle k actually visits node $i$, $Q_0^{uk}$=0 |

Based on the above notations, the VRPSPDMV can be formulated mathematically as follows: The notations of this study are given below:

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij}^u d_{ij}^u x_{kij}^u \tag{1}$$

$$\sum_{j \in N^u} x_{k0j}^u = 1 \quad \forall k \in K \tag{2}$$

*st.*

$$\sum_{i\in N^u} x^u_{ki0} = 1 \quad \forall k \in K \tag{3}$$

$$\sum_{k\in K}\sum_{i\in N^u} x^u_{kij} = 1 \quad \forall j \in D \tag{4}$$

$$\sum_{k\in K}\sum_{j\in N^u} x^u_{kij} \le 1 \quad \forall i \in P^u \tag{5}$$

$$\sum_{k\in K}\sum_{j\in P^u_i} z^u_{kij} = 1 \quad \forall i \in D \tag{6}$$

$$z^u_{kip} = \sum_{j\in P^u_i} x^u_{kjp} \quad \forall i \in D, p \in P^u_i, k \in K \tag{7}$$

$$\sum_{i\in N^u} x^u_{kpi} - \sum_{j\in N^u} x^u_{kdj} = 0 \qquad \forall d \in D, p \in P^u_d, k \in K \tag{8}$$

$$S^{uk}_i + s^u_i + t_{ij} - S^{uk}_j \le 0 \quad \forall i \in D, j \in P^u_j, k \in K \tag{9}$$

$$\sum_{i\in N^u} x^u_{kih} - \sum_{j\in N^u} x^u_{khi} = 0 \quad \forall h \in Nodes^u, k \in K \tag{10}$$

$$e^u_i \le S^{uk}_i \le l^u_i \quad \forall i \in N^u, k \in K \tag{11}$$

$$S^{uk}_i + s^u_i + t^u_{ij} - M(1 - x^u_{kij}) \le S^{uk}_j \qquad \forall i, j \in N^u, k \in K \tag{12}$$

$$0 \le Q^{uk}_i \le Q \qquad \forall i \in N^u, k \in K \tag{13}$$

$$Q^{uk}_i + q^u_j - M(1 - x^u_{kij}) \le Q^{uk}_j \qquad \forall i, j \in N^u, k \in K \tag{14}$$

$$\sum_{k\in K}\sum_{j\in N^u} x^u_{k0j} \le K_{num} \tag{15}$$

$$z^u_{kij} \in \{0,1\} \qquad \forall i \in D, j \in P^u_i, k \in K \tag{16}$$

$$x^u_{kij} \in \{0,1\} \quad \forall i, j \in N^u, i \ne j, k \in K \tag{17}$$

The objective function (1) states the objective of the VRPSPDMV, which is to minimize the total travel cost of the vehicle. Constraints (2) and constraints (3) guarantee that the route of each vehicle starts and ends at the depot. Constraints (4) ensure that each delivery node is must be visited exactly once by a single vehicle.Constraints (5) ensure that each pickup node is visited

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at most once by a single vehicle. Constraints (6) indicate that each delivery node has only one pickup node in the total pickup set capable of supplying this delivery node. Constraints (7) indicate that once a vehicle chooses a pickup node to supply the delivery node, the vehicle must have a path from other node to this pickup node. Constraints (8) indicate that the selected pickup node and delivery node are visited by the same vehicle. Constraints (9) indicate that the selected pickup node is visited before the delivery node. Constraints (10) are flow conservations constraints. Constraints (11) indicate time windows for all nodes. Constraints (12) ensure that $S_i^{uk}$ is set correctly along the paths. Constraints (13) ensure that the vehicle load is not greater than the vehicle capacity at any node. Constraints (14) ensure that the load variable is set correctly along the paths. Constraints (15) indicate that the number of vehicles starting from the depot must not exceed the initial number of vehicles. Constraints (16) and constraints (17) indicate binary variables.

## 4. Experimental Results

**Table 2.** The notations of VRP1PD1V

| | |
|---|---|
| $K_{num}$ | the initial number of vehicle given |
| 0 | depot |
| $n$ | the number of delivery nodes |
| $m$ | the number of pickup nodes |
| $i, j, h,p,d$ | id of node |
| $k$ | id of vehicle |
| $c_{ij}$ | the cost per unit distance from node i to node j, $c_{ij}$=1 |
| $d_{ij}$ | the Euclidean distance from $i$ to $j$ |
| $t_{ij}$ | the traveling time from $i$ to $j$ |
| $s_i$ | the loading or unloading duration of the node $i$ |
| $[e_i, l_i]$ | the earliest and latest start time for loading and unloading of node $i$ |
| $q_i$ | the demand of node $i$, $q_i > 0$ means that the goods are loaded at this node, $q_i < 0$ means that the goods are unloaded at this node, and the demand of depot is 0 |
| $Q$ | the max capacity of the vehicle |
| $M$ | an arbitrarily large positive integer |
| $D$ | set of delivery nodes, $D:=\{1,2,...,n\}$ |
| $P$ | set of pickup nodes, $P:=\{n+1, n+2, ..., n+m\}$ |
| $P_i$ | set of pickup nodes |
| $Nodes$ | set of all nodes except depot, $Nodes:=\{P \cup D\}$ |
| $N$ | all nodes, $N:=\{0 \cup P^u \cup D\}$ |
| $K$ | set of vehicles, $K:=\{1,2,...,K_{num}\}$ |
| $x_{kij}$ | is 1 when the vehicle $k$ travels from node $i$ to node $j$, otherwise is 0 |
| $S_i^k$ | time of vehicle $k$ when vehicle k actually visits node $i$, $S_0^k$=0 |
| $Q_i^k$ | capacity of vehicle $k$ when vehicle k actually visits node $i$, $Q_0^k$=0 |

This study uses CPLEX to solve the mathematical model. Let $c_{ij}^u$=1 in the model, then the objective function of is simplified as $\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} d_{kij}^u x_{kij}^u$ . All computational experiments were

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carried out on a Lenovo notebook with an Intel Core i7-6500U CPU@2.5 GHz. The model is implemented using the CPLEX OPL library and included in a C++ framework. Set the maximum running time of CPLEX to 3600s. And in order to verify the correctness and efficiency of the mathematical model, the model proposed in this paper is compared with the calculation results of the vehicle routing problem with one pickup node and delivery and single visit(VRP1PD1V). The notations of VRP1PD1V are given in Table 2.

Based on the above notations, the VRP1PD1V can be formulated mathematically as follows:

$$\min \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij} d_{ij} x_{kij} \tag{18}$$

$$st. \quad \sum_{j\in N} x_{k0j} = 1 \quad \forall k\in K \tag{19}$$

$$\sum_{i\in N} x_{ki0} = 1 \quad \forall k\in K \tag{20}$$

$$\sum_{k\in K}\sum_{i\in N} x_{kij} = 1 \quad \forall j\in Nodes \tag{21}$$

$$\sum_{i\in N} x_{kpj} - \sum_{j\in N} x_{kdj} = 0 \qquad \forall d\in D, p\in P_d, k\in K \tag{22}$$

$$S_i^k + s_i + t_{ij} - S_j^k \le 0 \qquad \forall i\in P_j, j\in D, k\in K \tag{23}$$

$$\sum_{i\in N} x_{kih} - \sum_{j\in N} x_{khj} = 0 \qquad \forall h\in Nodes, k\in K \tag{24}$$

$$e_i \le S_i^k \le l_i \qquad \forall i\in N, k\in K \tag{25}$$

$$S_i^k + s_i + t_{ij} - M(1 - x_{kij}) \le S_j^k \quad \forall i,j\in N, k\in K \tag{26}$$

$$0 \le Q_i^k \le Q \quad \forall i\in N, k\in K \tag{27}$$

$$Q_i^k + q_i - M(1 - x_{kij}) \le Q_j^k \quad \forall i,j\in N, k\in K \tag{28}$$

$$\sum_{k\in K}\sum_{i\in N} x_{k0j} \le K_{num} \tag{29}$$

$$x_{kij} \in \{0, 1\} \quad \forall i,\ j\in N, i\ne j, k\in K \tag{30}$$

The objective function (18) states the objective of the VRP1PD1V, which is to minimize the total travel cost of the vehicle. And let $c_{ij}$=1 in (18). Constraints (19) and constraints (20) guarantee that the route of each vehicle starts and ends at the depot. Constraints (21) ensure that all nodes must be visited exactly once by a single vehicle. Constraints (22) indicate that the corresponding pickup node and delivery node are visited by the same vehicle. Constraints (23) indicate that the pickup node is visited before the delivery node when serving the request.

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Constraints (24) are flow conservations constraints. Constraints (25) indicate time windows for all nodes. Constraints (26) ensure that $S_i^k$ is set correctly along the paths. Constraints (27) ensure that the vehicle load is not greater than the vehicle capacity at any node. Constraints (28) ensure that the load variable is set correctly along the paths. Constraints (29) indicate that the number of vehicles starting from the depot must not exceed the initial number of vehicles. Constraints (30) indicate binary variables.

## 4.1. Tuning Instances

There is no existing data set for VRPSPDMV. according to the model of VRPSPDMV, new data set are generated based on set data in Li and Lim[21]. The rules for data sets generated are as follows: (1) According to the original calculation example, select a certain proportion of delivery nodes randomly (controlled by parameters *removePrate*), and then delete the corresponding pickup nodes of these delivery nodes from the distribution network, and then select a pickup node randomly from the existing pickup nodes as the pickup node of the current deleted pickup node. (2) On the basis of the data sets generated in the first step, a certain proportion of delivery nodes (controlled by parameters *chooseDrate*) are randomly selected. For this part of delivery nodes, the amount of $a$ ($a \in [0,2]$) pickup nodes, which is not repeated, is selected as selective pickup nodes from the current set of pickup nodes. At this point, for the selected part of the delivery node, each delivery node has $a$+1 optional pickup nodes (the other 1 refers to the corresponding pickup node get in the first step) to fulfill the demand of delivery node. (3) Adjust the time window of all nodes to make them have feasible solutions. (4) Set the maximum vehicle load as 200, and set the initial number of vehicles as the number of delivery nodes. Set the number of delivery points of 5 and 10 respectively. And then the data sets of VRPSPDMV are generated. (5) Based on VRPSPDMV, reduce the number of pickup nodes corresponding to delivery node to one, and the data set of VRP1PD1V are generated.

Based on the above rule, a total of 24 data set of 5 and 10 delivery nodes of VRPSPDMV are generated and a total of 24 data set of 5 and 10 delivery nodes of VRP1PD1V are also generated.

## 4.2. Results

The results obtained by solving VRPSPDMV and VRP1PD1V respectively by CPLEX are shown as follows:

Table 3 is the results obtained on VRPSPDMV solved by CPLEX. In Table 3, n is the number of delivery nodes, *m* is the number of pickup nodes, and *m* in the Table 3 is the number of pickup nodes after spliting. "2" means the tarting node(depot) and terminal(depot) of the vehicle. $n + m + 2$ is the number of all nodes in the distribution network. *C* represents the cost obtained by solution, *K* represents the number of vehicles obtained by solution, *T* represents the solution time, *Gap* represents the difference between the upper and lower bounds of feasible integer solution of objective function obtained by CPLEX, and when *Gap* is 0, the optimal solution is obtained. "—"means feasible solution can not be obtained in the 3600s solved by CPLEX.

Table 4 is the results obtained on VRP1PD1V solved by CPLEX and the Comparison Between VRPSPDMV and VRP1PD1V. In Table 4, *n* is the number of delivery nodes of VRP1PD1V, *m* is the number of pickup nodes of VRP1PD1V. "2" means the tarting node(depot) and terminal(depot) of the vehicle. $n + m + 2$ is the number of all nodes in the distribution network. *C* represents the cost obtained by solution, *K* represents the number of vehicles obtained by solution, *T* represents the solution time, *Gap* represents the difference between the upper and lower bounds of feasible integer solution of objective function obtained by CPLEX, and when *Gap* is 0, the optimal solution is obtained. In addition, Table 4 shows the comparison between the results obtained by solving VRPSPDMV and VRP1PD1V using CPLEX. $Gap_N$ is the difference of the number of nodes in VRPSPDMV and VRP1PD1V, $N$ is the numbers of all nodes,

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$Gap_N$=($N_{VRP1PD1V}$-$N_{VRPSPDMV}$)/$N_{VRP1PD1V}$, $Gap_C$ is the best solution difference between VRPSPDMV and VRP1PD1V, $Gap_C$=($C_{VRPSPPD}$-$C_{VRPSPDMV}$)/$C_{VRP1PD1V}$. $Gap_K$ is the difference of number of vehicles between VRPSPDMV and VRP1PD1V, $Gap_K$=$K_{VRP1PD1V}$-$K_{VRPSPDMV}$.

**Table 3.** Results Obtained on VRPSPDMV solved by CPLEX

| **Instance** | ***n + m + 2*** | ***C*** | ***K*** | ***T*(sec)** | ***Gap*(%)** |
|---|---|---|---|---|---|
| lc101-5 | 5 + 9 + 2 | 169.18 | 1 | 1.26 | 0.00 |
| lc102-5 | 5 + 8 + 2 | 140.66 | 1 | 2.51 | 0.00 |
| lc201-5 | 5 + 8 + 2 | 135.54 | 1 | 5.35 | 0.00 |
| lc202-5 | 5 + 8 + 2 | 175.04 | 1 | 1.53 | 0.00 |
| lr101-5 | 5 + 7 + 2 | 220.34 | 2 | 1.72 | 0.00 |
| lr102-5 | 5 + 7 + 2 | 222.22 | 2 | 1.34 | 0.00 |
| lr201-5 | 5 + 9 + 2 | 246.53 | 2 | 5.77 | 0.00 |
| lr202-5 | 5 + 7 + 2 | 268.93 | 1 | 6.48 | 0.00 |
| lrc101-5 | 5 + 8 + 2 | 292.86 | 2 | 1.41 | 0.00 |
| lrc102-5 | 5 + 8 + 2 | 349.99 | 2 | 1.61 | 0.00 |
| lrc201-5 | 5 + 8 + 2 | 300.59 | 1 | 14.96 | 0.00 |
| lrc202-5 | 5 + 7 + 2 | 223.06 | 1 | 3.42 | 0.00 |
| lc101-10 | 10 + 15 + 2 | 428.14 | 3 | 3600.00 | 17.99 |
| lc102-10 | 10 + 13 + 2 | 438.90 | 3 | 3600.00 | 31.50 |
| lc201-10 | 10 + 14 +2 | 281.42 | 2 | 184.10 | 0.00 |
| lc202-10 | 10 + 13 + 2 | 473.07 | 1 | 3600.00 | 11.00 |
| lr101-10 | 10 + 17 + 2 | 263.74 | 3 | 3600.00 | 40.68 |
| lr102-10 | 10 + 13 + 2 | 392.34 | 3 | 3600.00 | 36.98 |
| lr201-10 | 10 + 17 + 2 | 355.30 | 2 | 1236.20 | 0.00 |
| lr202-10 | 10 + 18 + 2 | 339.53 | 2 | 3600.00 | 34.10 |
| lrc101-10 | 10 +16 + 2 | — | — | 3600.00 | — |
| lrc102-10 | 10 + 13 + 2 | 673.98 | 4 | 3600.00 | 51.94 |
| lrc201-10 | 10 + 17 + 2 | 364.77 | 2 | 3600.00 | 24.16 |
| lrc202-10 | 10 + 15 + 2 | 272.90 | 1 | 3600.00 | 8.55 |

As can be seen from Table 3, the results obtained by using CPLEX to solve VRPSPDMV. For the data set with 5 delivery nodes, the scale of all nodes is between 14 and 16, which can be solved by CPLEX, and the optimal solutions of all instances can be obtained within 3600s. For the data set with 10 delivery nodes, the total number of nodes in the network is between 25 and 30. Only two instances can get the optimal solution, which are lc201-10 and lr201-10. Eight instances Eight examples can get feasible solutions in 3600s. There are two instances, lc102-10 and lrc101-10, which cannot obtain the feasible solution in 3600s.

As can be seen from Table 4, the results obtained by using CPLEX to solve VRP1PD1V show that for the data set with 5 delivery nodes, the total number of nodes in the network is 10, and for the data set with 10 delivery nodes, the total number of nodes in the network is between 17 and 18. The optimal solutions of all VRP1PD1V instances are obtained by CPLEX and the solution time to solve is quickly.

**Table 4.** Results Obtained on VRP1PD1V and the Comparison between VRPSPDMV and VRP1PD1V

| Instance | Results of VRP1PD1V | | | | | Comparison | | |
|---|---|---|---|---|---|---|---|---|
| | *n + m + 2* | *C* | *K* | *T*(sec) | *Gap*(%) | $Gap_N$ (%) | $Gap_C$ (%) | $Gap_K$ |
| lc101-5 | 5 + 3 + 2 | 169.18 | 1 | 0.16 | 0.00 | -60.00 | 0.00 | 0 |
| lc102-5 | 5 + 3 + 2 | 164.20 | 2 | 0.26 | 0.00 | -50.00 | 14.34 | 1 |
| lc201-5 | 5 + 3 + 2 | 135.54 | 1 | 0.17 | 0.00 | -50.00 | 0.00 | 0 |
| lc202-5 | 5 + 3 + 2 | 216.50 | 1 | 0.30 | 0.00 | -50.00 | 19.15 | 0 |
| lr101-5 | 5 + 3 + 2 | 249.00 | 3 | 0.23 | 0.00 | -40.00 | 11.51 | 1 |
| lr102-5 | 5 + 3 + 2 | 224.04 | 2 | 0.14 | 0.00 | -40.00 | 0.81 | 0 |
| lr201-5 | 5 + 3 + 2 | 246.53 | 2 | 0.39 | 0.00 | -60.00 | 0.00 | 0 |
| lr202-5 | 5 + 3 + 2 | 269.70 | 1 | 0.28 | 0.00 | -40.00 | 0.29 | 0 |
| lrc101-5 | 5 + 3 + 2 | 297.29 | 2 | 0.36 | 0.00 | -50.00 | 1.49 | 0 |
| lrc102-5 | 5 + 3 + 2 | 377.21 | 3 | 0.12 | 0.00 | -50.00 | 7.21 | 1 |
| lrc201-5 | 5 + 3 + 2 | 300.59 | 1 | 0.23 | 0.00 | -50.00 | 0.00 | 0 |
| lrc202-5 | 5 + 3 + 2 | 247.35 | 1 | 0.17 | 0.00 | -40.00 | 9.82 | 0 |
| lc101-10 | 10 + 6 + 2 | 495.46 | 4 | 125.25 | 0.00 | -50.00 | 13.59 | 1 |
| lc102-10 | 10 + 5 + 2 | 489.62 | 3 | 478.42 | 0.00 | -47.06 | 10.36 | 0 |
| lc201-10 | 10 + 6 + 2 | 281.42 | 2 | 8.08 | 0.00 | -44.44 | 0.00 | 0 |
| lc202-10 | 10 + 6 + 2 | 474.24 | 1 | 42.17 | 0.00 | -38.89 | 0.25 | 0 |
| lr101-10 | 10 + 5 + 2 | 263.62 | 1 | 12.56 | 0.00 | -70.59 | -0.05 | -2 |
| lr102-10 | 10 + 5 + 2 | 391.49 | 3 | 51.77 | 0.00 | -47.06 | -0.22 | 0 |
| lr201-10 | 10 + 5 + 2 | 389.79 | 1 | 4.45 | 0.00 | -70.59 | 8.85 | -1 |
| lr202-10 | 10 + 6 + 2 | 358.12 | 1 | 3475.38 | 0.00 | -66.67 | 5.19 | -1 |
| lrc101-10 | 10 + 5 + 2 | 390.86 | 4 | 5.09 | 0.00 | -64.71 | — | — |
| lrc102-10 | 10 + 6 + 2 | 600.13 | 4 | 614.86 | 0.00 | -38.89 | -12.31 | 0 |
| lrc201-10 | 10 + 6 + 2 | 408.47 | 1 | 164.49 | 0.00 | -61.11 | 10.70 | -1 |
| lrc202-10 | 10 + 5 + 2 | 544.50 | 5 | 22.71 | 0.00 | -58.82 | 49.88 | 4 |

By comparing the results of the two models in Table 4, it can be seen that compared with VRP1PD1V, VRPSPDMV has more nodes in the network. It is more difficult to solve VRPSPDMV, and CPLEX takes more time to solve VRPSPDMV. For instances that can get the optimal solution of VRPSPDMV, the cost of the optimal solution of VRPSPDMV is less than VRP1PD1V, and the number of vehicles is no more than VRP1PD1V. For instances that can only obtain the feasible solution of VRPSPDMV, the cost of the solution obtained by VRPSPDMV is less and the number of vehicles used is less. By comparing the results between the two models, it can be seen that although VRPSPDMV has more total nodes, greater complexity and longer solving time than VRP1PD1V, VRPSPDMV can reduce the distribution cost and the number of distribution vehicles to a certain extent comparing with VRP1PD1V.

## 5. Conclusion

Based on the actual logistics distribution scenario in omni-channel retailing, this paper proposes a new variant model of VRPPD, the new problem considers the characteristics of time window, the selection of pickup nodes and multiple visits to pickup nodes. Based on the classical data set in related research, a set of new data set of new model are generated. In this paper, CPLEX is used to solve the new problem.

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In order to verify the effectiveness of the model, this paper compares the proposed model with the vehicle routing problem with one pickup node and delivery and single visit. The results show that the new model is more complex and more time-consuming to solve, but it can reduce the distribution cost of the distribution network and the number of vehicles used. The research can provide advice for the optimization of distribution newtwork in omni-channel retailing.

The optimization solver CPLEX can only solve small-scale data sets in an acceptable time. The data set solved in this paper are small in scale, and heuristic algorithms will be developed to solve large-scale data set of the model in the following research.

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