

Reliability Assessment of a Stochastic Flow Delivery Network under Spoilage Consideration

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Abstract

From the performance assessment perspective, it is of central importance to assess the reliability of a stochastic flow distribution network (SFDN) in which each node represents a supplier, a transfer center, or a market, and each route joining a pair of nodes, in addition to multi-valued capacities, is featured with a spoilage rate. As a consequence, network reliability is the probability that the SFDN is able to distribute the required quantity of goods to meet the market demand under delivery spoilage considerations. A minimal paths (MPs) based algorithm is presented to calculate the network reliability, together with an example to illustrate the procedure. A real fruit distribution network is accordingly discussed to demonstrate the utility of the algorithm and the managerial implication of network reliability.

Keywords

Reliability; Stochastic Flow Distribution Network; Delivery Spoilage; Minimal Paths.

1. Introduction

Many real-life networks can be modeled as stochastic flow networks, such as manufacturing networks, electric power networks, computer networks, or logistics networks [1]. In the logistics networks, transportation activity plays an essential role. Transportation activity involves the processes of product manufacturing, transportation, storage, and sale. Many companies utilize product attribute, market position, technology, regional culture, labor and policy to design their transportation network [2]. A transportation network is composed of a series of nodes and routes, where each route represents a supplier, a transfer center, or a market and each route joining a pair of nodes can represent the logistics activity [1]. There is a carrier on each route responsible for the delivery service on this section of the road. In the real world, the available delivery capacity of each carrier is uncertain. That is, the available delivery capacity which is called the number of containers may be partially reserved by other customers who are not the members in the logistics network [3]. Thus, the available delivery capacity of a carrier has multiple states and can be represented by a probability distribution which can be obtained from the carrier's database [1, 4-7]. We regard the commodities transported by the logistics network as the flow. Thus, any logistics network can be regarded as a stochastic flow delivery network (SFDN).

In reality, there are some perishable commodities, such as meat, milk, fruit and eggs. It is easy to decay or be spoiled in the course of delivery due to time, collisions, air temperature, natural disasters, traffic accidents and other factors. It is not uncommon for damage to occur in various network systems. Rong et al. presented a method to model food quality degradation during production and distribution planning [8]. Taking transport damage during transportation into account, Keizer et al. focused on designing the logistics network for perishable products with heterogeneous quality decay [9]. In the context of transport damage, the intact products arriving at destinations may be cannot meet the market needs. Then, some researchers shifted

attention to the network reliability with goods deterioration. Lin et al. defined the reliability of a distribution network with goods deterioration as the probability that the network is able to meet the market demand for intact commodities under delivery damage and budget limit considerations. Postulate that each minimal path (MP) is associated with a spoilage rate, Lin et al. first described the assignment strategy of product flow on each MP so that the market demand for intact products can be met, and then proposed an algorithm to evaluate the network reliability [1].

In this article, we also Take SFDN reliability under transportation deterioration considerations into account, which is defined as the probability that the SFDN is able to successfully transport the required quantities of products from the single supplier to multiple destinations under transport damage consideration. To better illustrate the relationship of MPs, we associate a specific damage rate with each arc, instead of each MP. In this case, the change of damage rate of one arc may influence damage rates of several MPs, which is more practical in the transportation process. And a method is proposed to evaluate the SFDN reliability in the bases of feasible flow patterns which are minimal capacity vectors satisfying market demand under transport damage consideration. The proposed method utilizes MPs and market demands to obtain feasible flow patterns. Finally, a simple network is described to illustrate the process of the method and illustrate the management implication of network reliability.

2. Construction of the SFDN Model

This section describes the relationships between the transportation flow, transportation capacity, transportation damage and demand, and then an SFDN model is developed. Before developing the SFDN model, the notations and assumptions are introduced in the following subsection.

2.1. Notations and Assumptions

All notations used in this paper are summarized in the following list. To be worthy of attention, there exists a contracted carrier alone arc a_i in (V, E) to be responsible for freight delivery, $i = 1, 2, \dots, n$. The available capacity of each contracted carrier is a random variable noted by h_{ij} , and j takes integer values from 1 to π_i according to a given probability distribution.

Table 1. Notation list

V : set of nodes.
E : set of arcs connecting nodes.
(V, E) : a stochastic-flow distribution network.
n : number of arcs.
a_i : the i th route in (V, E) , $i = 1, 2, \dots, n$.
s : supplier, i.e., source node.
m : number of markets.
t_e : the e th market in (V, E) , i.e., sink node, $e=1, 2, \dots, m$.
π_i : number of states that route a_i owns.
h_{ij} : The j th available capacity of each contracted carrier along route a_i , $j = 1, 2, \dots, \pi_i$.
W : the maximal capacity vector of (V, E) .
w : the capacity required for each unit of commodity.
z_e : the number of minimal path (MP) connecting source s and sink t_e , $e=1, 2, \dots, m$.
$MP_{e,j}$: the MP connecting source s and sink t_e , $j = 1, 2, \dots, z_e$.
d_e : the demand of market t_e .

<p>D: the market demand vector: (d_1, d_2, \dots, d_m).</p> <p>$f_{e,j}$: the ideal flow through $MP_{e,j}$, $e=1, 2, \dots, m, j = 1, 2, \dots, z_e$.</p> <p>$F$: the ideal flow vector: $(f_{1,1}, f_{1,2}, \dots, f_{1,z_1}, f_{2,1}, f_{2,2}, \dots, f_{2,z_2}, \dots, f_{e,1}, f_{e,2}, \dots, f_{e,z_e}, \dots, f_{m,1}, f_{m,2}, \dots, f_{m,z_m})$.</p> <p>$p_i$: delivery spoilable rate of arc a_i.</p> <p>$p_{e,j}$: delivery spoilable rate of $MP_{e,j}$, $e=1, 2, \dots, m, j = 1, 2, \dots, z_e$.</p> <p>$P$: The set of delivery spoilable rate of arc a_i, $P = (p_1, p_2, \dots, p_n)$.</p> <p>$o_{e,j}$: Actual delivered flow through $MP_{e,j}$.</p> <p>I: The actual flow vector: $(o_{1,1}, o_{1,2}, \dots, o_{1,z_1}, o_{2,1}, o_{2,2}, \dots, o_{2,z_2}, \dots, o_{e,1}, o_{e,2}, \dots, o_{e,z_e}, \dots, o_{m,1}, o_{m,2}, \dots, o_{m,z_m})$</p> <p>$x_i$: Current available capacity of a_i, $i = 1, 2, \dots, n$</p> <p>X: Current capacity vector of (V, E, W), $X = (x_1, x_2, \dots, x_n)$</p> <p>$R_{D,P}$: Network reliability</p> <p>MCV: minimal carrying capacity vector</p> <p>(D, P)-MCVs: the minimal carrying capacity vectors that can satisfy the demand vector D under the spoilage pattern P.</p> <p>Ψ: Set of minimal capacity vector feasible under (D, P).</p> <p>$\lceil x \rceil$: the smallest integer which is larger than or equal to x.</p> <p>$\lfloor x \rfloor$: the largest integer which is less than or equal to x.</p>
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The following assumptions are made in this article:

- (1) All flows in the network satisfy the flow-conservation law, i.e., total flows into and from a node (other than the source and sink nodes) are all equal.
- (2) Flow in the network is an integer value.
- (3) The transportation capacities of various logistics service provider are statistically independent.

2.2. Ideal Flow Vector and Actual Flow Vector

let $F = (f_{1,1}, f_{1,2}, \dots, f_{1,z_1}, f_{2,1}, f_{2,2}, \dots, f_{2,z_2}, \dots, f_{e,1}, f_{e,2}, \dots, f_{e,z_e}, \dots, f_{m,1}, f_{m,2}, \dots, f_{m,z_m})$ be an ideal flow vector, with $f_{e,j}$ denoting the ideal flow (the number of transported freights without considering transportation damage) traveling through $MP_{e,j}$; it should be an integer value according to assumption II. All flows in the network satisfy the flow-conservation law, i.e., total flows into and from a node (other than the source and destination nodes) are all equal. The demand vector is denoted by $D = (d_1, d_2, \dots, d_m)$, where d_e is the required units of commodity for the market t_e . Hence, any ideal flow vector F is said to meet the exact demand vector D under the flow conservation principle if and only if it satisfies the following constraint:

$$\sum_{j=1}^{z_e} f_{e,j} = d_e \quad \text{for } e=1,2,\dots,m. \tag{1}$$

Where the term $\sum_{j=1}^{z_e} f_{e,j}$ represents the total amount of flow through (V, E) , such an F not taking damage consideration into transportation process yet is called an ideal flow vector. However, the products may be damaged due to collisions, traffic accident, natural disaster, whether, time during delivery. Hence the ideal flow vector may un-satisfy the market requirement. Generally speaking, an average ratio of the number of damaged products to the number of total products for a route a_i can be obtained by long-term observation. Such a ratio denoted by p_i is called the damage rate of route a_i . Why we consider the delivery damage of

route a_i instead of the delivery damage of minimal path $MP_{e,j}$? The reason is that when consideration of the delivery damage of $MP_{e,j}$, the delivery damage rate of each path is uncorrelated and independent of each other, but a route generally connects more than one path. For example, in the sensitivity analysis of damage rate, the change of the delivery damage rate of one route may affect the value of the delivery damage rate of several minimal paths. Therefore, the consideration of the delivery damage of route a_i enhances the correlation degree of each $MP_{e,j}$ and it is closer to the real life. In addition, the notation $p_{e,j}$ is defined as the damage rate of a specified $MP_{e,j}$. Each intact flow traveling through $MP_{e,j}$ can be calculated by $f_{e,j} \times (1 - p_{e,j})$. The damage rate $p_{e,j}$ of a specified $MP_{e,j}$ can be derived from the damage rate of route a_i according to the following equation:

$$p_{e,j} = 1 - \prod_{a_i \in MP_{e,j}} (1 - p_i) \text{ for } e = 1, 2, \dots, \lambda \text{ and } j = 1, 2, \dots, z_e. \tag{2}$$

For example, a network consisting of one supplier, one transfer center, and two markets has two MPs: $MP_{1,1} = \{a_1, a_2\}$ and $MP_{2,1} = \{a_1, a_3\}$ (refer to Fig. 1). For the demand $d_1 = 2$ and $d_2 = 1$, the ideal-flow vector is $F = (2, 1)$. If the damage rate of route a_1, a_2 , and a_3 is $p_1 = 0.08, p_2 = 0.05$, and $p_3 = 0.06$ respectively, we can get $p_{1,1} = 1 - (1 - 0.08)(1 - 0.05) = 0.126$ and $p_{2,1} = 1 - (1 - 0.08)(1 - 0.06) = 0.1352$. The number of intact flows traveling to the market t_1 is $f_{1,1} \times (1 - p_{1,1}) = 2 \times (1 - 0.126) = 1.748 < d_1$, the number of intact flows traveling to the market t_2 is $f_{2,1} \times (1 - p_{2,1}) = 1 \times (1 - 0.1352) = 0.8648 < d_2$.

In order to make the number of goods delivered to the market t_e meet the demand d_e , the notation $I = (o_{1,1}, o_{1,2}, \dots, o_{1,z_1}, o_{2,1}, o_{2,2}, \dots, o_{2,z_2}, \dots, o_{e,1}, o_{e,2}, \dots, o_{e,z_e}, \dots, o_{m,1}, o_{m,2}, \dots, o_{m,z_m})$ is represented as an actual-delivered flow vector, where $o_{e,j}$ is the actual delivered flow which is derived from the ideal flow $f_{e,j}$ via the following equation:

$$o_{e,j} = \lceil f_{e,j} / (1 - p_{e,j}) \rceil \tag{3}$$

Following the above example, $o_{1,1} = \lceil 2 / (1 - 0.126) \rceil = \lceil 2.288 \rceil = 3$ and $o_{2,1} = \lceil 1 / (1 - 0.1352) \rceil = \lceil 1.156 \rceil = 2$, where “2.288” and “1.156” signify the smallest values that satisfy the market demand based on the ideal-flow vector $F = (2, 1)$ when transportation damage is taken into account. According to assumption II, the actual-delivered flow must be an integer value. since we consider the smallest integer value 3 which is larger than or equal to 2.288 as the actual-delivered flow of $MP_{1,1}$. If $o_{1,1} < 2.288$, such as 2, the number of intact goods delivered to the market t_1 is $\lfloor 2 \times (1 - 0.126) \rfloor = \lfloor 1.748 \rfloor = 1$ and does not satisfy $d_1 = 2$. obviously, each such I is the smallest flow vector to meet the demand vector D with transportation damage consideration.

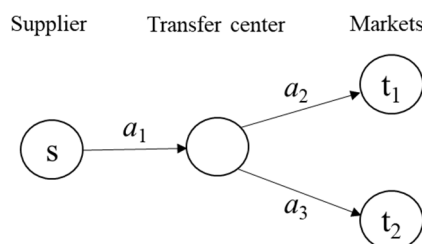


Fig 1. A network with a single supplier and two markets

2.3. Actual Flow Vector and Delivery Capacity Vector

Let $X = (x_1, x_2, \dots, x_n)$ be a delivered capacity vector, where x_i denotes the current delivery capacity of route a_i and takes an integer value $h_{i1}=0, h_{i2}, \dots,$ or $h_{i\pi_i}$ for $i = 1, 2, \dots, n$. The maximum delivery-capacity vector of (V, E) is denoted by $W = (h_{1\pi_1}, h_{2\pi_2}, \dots, h_{n\pi_n})$. For convenience, let I_W denote the set of actual flow vectors feasible under W . The w is the consumed delivery capacity per unit of flow. Hence, any actual flow vector I is said to be feasible under W if and only if it satisfies the following constraints:

$$\left[w \sum_{e=1}^m \sum_{j: a_i \in MP_{e,j}} o_{e,j} \right] \leq h_{i\pi_i} \text{ for } i=1, 2, \dots, n \tag{4}$$

$\left[w \sum_{e=1}^m \sum_{j: a_i \in MP_{e,j}} o_{e,j} \right]$ is the consumed transportation capacity of the total flow through a_i . constraint (4) represents the consumed transportation capacity on route a_i cannot exceed the maximal transportation capacity of a_i for $i = 1, 2, \dots, n$.

Equally, any I satisfying the constraint (5) is said to be feasible under X .

$$\left[w \sum_{e=1}^m \sum_{j: a_i \in MP_{e,j}} o_{e,j} \right] \leq x_i \text{ for } i=1, 2, \dots, n. \tag{5}$$

Constraint (5) represents the consumed capacity on route a_i cannot exceed the current capacity state of route a_i and I_X is the set of all actual-flow vectors feasible under X .

2.4. SFDN Reliability Evaluation

SFDN reliability denoted by $R_{D,P}$ is defined as the probability that the SFDN can satisfy the demand vector D under the spoilage pattern P and can successfully transport d_e units of goods from the supplier s to the market t_e for $e=1, 2, \dots, m$. That is, $R_{D,P} \equiv \Pr\{X|X \text{ fulfils the requirement } (D, P)\}$. For convenience, let $\Psi \equiv \{X|X \text{ fulfils } (D, P)\}$. Then, the SFDN reliability $R_{D,P}$ is

$$R_{D,P} = \Pr(\Psi) = \sum_{X \in \Psi} \Pr(X) \tag{6}$$

where $\Pr(X) = \Pr\{x_1\} \times \Pr\{x_2\} \times \dots \times \Pr\{x_n\}$ by assumption 3.

2.5. Define (D, P)-MCV to Calculate SFDN Reliability

one way to obtain $R_{D,P}$ is to enumerate all $X \in \Psi$ and then sum up their probability. However, it is not an efficient way and it will become time-consuming as the network becomes larger. Instead, the study utilized the concept of minimal carrying capacity vectors (MCVs) to increase the computational efficiency for the SFDN reliability evaluation. For convenience, let (D, P) -MCV denote such an MCV. Before defining the (D, P) -MCV, the following two definitions related to the comparison between two delivery-capacity vectors are introduced.

Definition 3: $X \leq Y: (x_1, x_2, \dots, x_m) \leq (y_1, y_2, \dots, y_m)$ if and only if $x_i \leq y_i$ for each i .

Definition 4: $X < Y: (x_1, x_2, \dots, x_m) \leq (y_1, y_2, \dots, y_m)$ if and only if $X \leq Y$ and $x_i < y_i$ for at least one i .

Definition 5: Any MCV in Ψ is named a (D, P) -MCV. That is, if X is a (D, P) -MCV, then $Y \notin \Psi$ for any delivery-capacity vector Y with $Y < X$.

Suppose the total b (D, P) -MCVs are X_1, X_2, \dots, X_b . Then, Ψ can be represented as $\{\bigcup_{i=1}^b \{X \mid X \geq X_i\}\}$.

Thus the SFDN reliability can be rewritten as $R_{D,P} = \sum_{X \in \Psi} \Pr(X) = \Pr\{\bigcup_{i=1}^b \{X \mid X \geq X_i\}\}$. This probability can be calculated by the inclusion-exclusion principle [10] or the recursive sum of disjoint products (RSDP) [11]. Because the RSDP has better computational efficiency than others, especially for larger networks [12], this article adopts the RSDP to calculate $\Pr\{\bigcup_{i=1}^b \{X \mid X \geq X_i\}\}$. In order to generate all (D, P) -MCVs to calculate $R_{D,P}$, the features of (D, P) -MCV are described as follows.

2.6. Generate All (D, P) -MCVs

In this subsection, we try to generate all (D, P) -MCVs. For each actual flow vector $I \in I_W$, we generate the corresponding delivery-capacity vector X through the following equation:

$$X_i = \begin{cases} h_{i1} & \text{if } h_{i1} \geq \left[w \sum_{e=1}^m \sum_{j:a_i \in MP_{e,j}} o_{e,j} \right] \\ h_{ik} & \text{if } h_{ik} \geq \left[w \sum_{e=1}^m \sum_{j:a_i \in MP_{e,j}} o_{e,j} \right] > h_{ik-1}, k = \{2, 3, \dots, \pi_i\}, i = 1, 2, \dots, n \end{cases} \tag{7}$$

Such a transformation guarantees that $I \in I_X$ and the X fulfills (D, P) . In particular, $I \notin I_Y$ for any $Y < X$. That is, the transformed X satisfies the demand d with transportation damage P . Thus, any X transformed via equation (7) is treated as a (D, P) -MCV candidate.

2.7. The Algorithm for SFDN Reliability Evaluation

Based on the addressed SFDN model, an algorithm is proposed to evaluate the network reliability as follows.

Step 1. Find all ideal-flow vector F satisfying the following demand constraint:

$$\sum_{j=1}^{z_e} f_{e,j} = d_e \text{ for } e=1,2,\dots,m. \tag{8}$$

If there is no F satisfying the constraint, then $R_{D,P} = 0$ and quit the algorithm.

Step 2. Transform each ideal-flow vector F obtained from Step 1 into actual-flow vector I via

$$p_{e,j} = 1 - \prod_{a_i \in MP_{e,j}} (1 - p_i) \text{ for } e = 1, 2, \dots, \lambda \text{ and } j = 1, 2, \dots, z_e. \tag{9}$$

$$o_{e,j} = \lceil f_{e,j} / (1 - p_{e,j}) \rceil \tag{10}$$

Step 3. Utilize the following constraints to check whether each I from Step 2 is a feasible actual-flow vector under W to satisfy the budget B or not.

$$\left[w \sum_{e=1}^m \sum_{j:a_i \in MP_{e,j}} o_{e,j} \right] \leq h_{i\pi_i} \text{ for } i=1,2,\dots,n \tag{11}$$

If there is no I satisfying the constraints, then $R_{D,P} = 0$ and quit the algorithm.

Step 4. Transform each feasible actual-delivered flow vector I into X via the following equation:

$$X_i = \begin{cases} h_{i1} & \text{if } h_{i1} \geq \left[w \sum_{e=1}^m \sum_{j: a_i \in MP_{e,j}} o_{e,j} \right] \\ h_{ik} & \text{if } h_{ik} \geq \left[w \sum_{e=1}^m \sum_{j: a_i \in MP_{e,j}} o_{e,j} \right] > h_{ik-1}, k = \{2, 3, \dots, \pi_i\}, i = 1, 2, \dots, n \end{cases} \quad (12)$$

Each X is a (D, P) -MCV candidate.

Step 5. Adopt comparison algorithm to check whether X driven by Step 4 are (D, P) -MCVs.

Step 6. Utilize the RSDP method to compute $R_{D,R}$.

3. A Simple Example to Illustrate the Solution Procedure

A simple distribution network including a single supplier, two distribution centers, two markets, and six arcs (see figure 2) is utilized to show the SFDN reliability evaluation algorithm. The data related to available capacity and probability distribution is given in Table 2. There are four MPs : $MP_{1,1} = \{a_1, a_3\}$ and $MP_{1,2} = \{a_2, a_5\}$ connecting s and t_1 , $MP_{2,1} = \{a_1, a_4\}$ and $MP_{2,2} = \{a_2, a_6\}$ connecting s and t_2 . Suppose the damage rates are $p_1 = 0.06, p_2 = 0.1, p_3 = 0.02, p_4 = 0.03, p_5 = 0.02, p_6 = 0.01$. The required capacity per unit of flow is 0.6, i.e. $w = 0.6$. For $D = (3, 2)$, how to acquire all $((3, 2), P)$ -MCVs and how to evaluate the SFDN reliability are illustrated in the following steps.

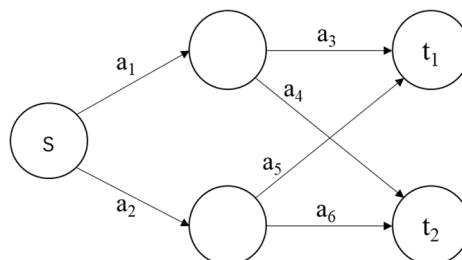


Fig 2. A simple distribution networks

Table 2. The logistics carrier's capacity data of each route

	available carrying capacity				
	$h_{i1}=0$	$h_{i2}=1$	$h_{i3}=2$	$h_{i4}=3$	$h_{i5}=4$
	Route (a_i)			Probability	
a_1	0.010 ^a	0.020	0.050	0.1000	0.820
a_2	0.010	0.050	0.050	0.080	0.810
a_3	0.010	0.010	0.050	0.930	0.000 ^b
a_4	0.010	0.010	0.980	0.000	0.000
a_5	0.005	0.005	0.020	0.970	0.000
a_6	0.010	0.020	0.970	0.000	0.000

^a the probability means $\Pr(h_{11}) = \Pr(x_1 = h_{11}) = \Pr(x_1 = 0) = 0.010$.

^b the logistics carrier does not provide the capacity.

Step 1. Find all F satisfying the following constraint:

$$f_{1.1} + f_{1.2} = 3$$

$$f_{2.1} + f_{2.2} = 2$$

This step generates 12 ideal flow vectors. For convenience, we list them in the first column of Table 3.

Step 2. Transform each F driven by Step 1 into I . For example, convert $F_1 = (0, 3, 0, 2)$ via

$$\begin{aligned} P_{1.1} &= 1 - (1 - 0.06) \times (1 - 0.02) = 0.0788, \\ P_{1.2} &= 1 - (1 - 0.1) \times (1 - 0.02) = 0.1180, \\ P_{2.1} &= 1 - (1 - 0.06) \times (1 - 0.03) = 0.0882, \\ P_{2.2} &= 1 - (1 - 0.1) \times (1 - 0.01) = 0.1090. \\ o_{1.1} &= \lceil 0 / (1 - 0.0788) \rceil = 0, \\ o_{1.2} &= \lceil 3 / (1 - 0.1180) \rceil = 4, \\ o_{2.1} &= \lceil 0 / (1 - 0.0882) \rceil = 0, \\ o_{2.2} &= \lceil 2 / (1 - 0.1090) \rceil = 3, \end{aligned}$$

to obtain $I_1 = (0, 4, 0, 3)$. All actual delivered flow vectors are given in the second column of Table 3.

Step 3. Employ the following constraints to check whether each I from Step 2 satisfies $W = (4, 4, 3, 2, 3, 2)$ or not.

$$\begin{aligned} 1 &\leq \lceil 0.6 \times o_{1.1} + 0.6 \times o_{2.1} \rceil \leq 4, \\ 1 &\leq \lceil 0.6 \times o_{1.2} + 0.6 \times o_{2.2} \rceil \leq 4, \\ 0 &\leq \lceil 0.6 \times o_{1.1} \rceil \leq 3, \\ 0 &\leq \lceil 0.6 \times o_{2.1} \rceil \leq 2, \\ 0 &\leq \lceil 0.6 \times o_{1.2} \rceil \leq 3, \\ 0 &\leq \lceil 0.6 \times o_{2.2} \rceil \leq 2, \end{aligned}$$

In total, we obtain 10 feasible actual delivered flow vectors and list them in the third column of Table 3.

Step 4. Convert each feasible actual delivered flow vector I into X via

$$\begin{aligned} x_1 &= \begin{cases} h_{11} & \text{if } h_{11} \geq \lceil 0.6 \times o_{1.1} + 0.6 \times o_{2.1} \rceil \\ h_{1k} & \text{if } h_{1k} \geq \lceil 0.6 \times o_{1.1} + 0.6 \times o_{2.1} \rceil > h_{1k-1}, k = \{2, 3, 4, 5\} \end{cases}, \\ x_2 &= \begin{cases} h_{21} & \text{if } h_{21} \geq \lceil 0.6 \times o_{1.2} + 0.6 \times o_{2.2} \rceil \\ h_{2k} & \text{if } h_{2k} \geq \lceil 0.6 \times o_{1.2} + 0.6 \times o_{2.2} \rceil > h_{2k-1}, k = \{2, 3, 4, 5\} \end{cases}, \\ x_3 &= \begin{cases} h_{31} & \text{if } h_{31} \geq \lceil 0.6 \times o_{1.1} \rceil \\ h_{3k} & \text{if } h_{3k} \geq \lceil 0.6 \times o_{1.1} \rceil > h_{3k-1}, k = \{2, 3, 4\} \end{cases}, \\ x_4 &= \begin{cases} h_{41} & \text{if } h_{41} \geq \lceil 0.6 \times o_{2.1} \rceil \\ h_{4k} & \text{if } h_{4k} \geq \lceil 0.6 \times o_{2.1} \rceil > h_{4k-1}, k = \{2, 3\} \end{cases}, \\ x_5 &= \begin{cases} h_{51} & \text{if } h_{51} \geq \lceil 0.6 \times o_{1.2} \rceil \\ h_{5k} & \text{if } h_{5k} \geq \lceil 0.6 \times o_{1.2} \rceil > h_{5k-1}, k = \{2, 3, 4\} \end{cases}, \\ x_6 &= \begin{cases} h_{61} & \text{if } h_{61} \geq \lceil 0.6 \times o_{2.2} \rceil \\ h_{6k} & \text{if } h_{6k} \geq \lceil 0.6 \times o_{2.2} \rceil > h_{6k-1}, k = \{2, 3\} \end{cases}, \end{aligned}$$

Then, we obtain 9 $((3, 2), P)$ -MCV candidates, $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$ and X_9 , listed in the fourth column of Table 3.

Step 5. The comparison algorithm generates 4 $((3, 2), P)$ -MCVs, X_2, X_5, X_6 and X_8 .

Step 6. The SFDN reliability $R_{((3,2),P)} = \Pr\{\bigcup_{i=2,5,6,8} \{X \mid X \geq X_i\}\} = 0.90582$ is calculated through the RSDP. That is, the probability of the SFDN to successfully transport $D = (3, 2)$ to the markets subject to $P = (0.06, 0.1, 0.02, 0.03, 0.02, 0.01)$ is 0.90582.

Table 3. The $((3, 2), P)$ -MCVs

Step 1	Step 2	Step 3	Step 4	Step 5
$F_1 = (0, 3, 0, 2)$	$I_1 = (0, 4, 0, 3)$	Unsatisfy $W = (4, 4, 3, 2, 3, 2)$	-	-
$F_2 = (0, 3, 1, 1)$	$I_2 = (0, 4, 2, 2)$	I_2 is feasible	$X_1 = (2, 4, 0, 2, 3, 2)$	No, $X_1 > X_2$
$F_3 = (0, 3, 2, 0)$	$I_3 = (0, 4, 3, 0)$	I_3 is feasible	$X_2 = (2, 3, 0, 2, 3, 0)$	Yes
$F_4 = (1, 2, 0, 2)$	$I_4 = (2, 3, 0, 3)$	I_4 is feasible	$X_3 = (2, 4, 2, 0, 2, 2)$	No, $X_3 > X_6$
$F_5 = (1, 2, 1, 1)$	$I_5 = (2, 3, 2, 2)$	I_5 is feasible	$X_4 = (3, 3, 2, 2, 2, 2)$	No, $X_3 > X_5$
$F_6 = (1, 2, 2, 0)$	$I_6 = (2, 3, 3, 0)$	I_6 is feasible	$X_5 = (3, 2, 2, 2, 2, 0)$	Yes
$F_7 = (2, 1, 0, 2)$	$I_7 = (3, 2, 0, 3)$	I_7 is feasible	$X_6 = (2, 3, 2, 0, 2, 2)$	Yes
$F_8 = (2, 1, 1, 1)$	$I_8 = (3, 2, 2, 2)$	I_8 is feasible	$X_4 = (3, 3, 2, 2, 2, 2)$	No, $X_4 > X_5$
$F_9 = (2, 1, 2, 0)$	$I_9 = (3, 2, 3, 0)$	I_9 is feasible	$X_7 = (4, 2, 2, 2, 2, 0)$	No, $X_7 > X_5$
$F_{10} = (3, 0, 0, 2)$	$I_{10} = (4, 0, 0, 3)$	I_{10} is feasible	$X_8 = (3, 2, 3, 0, 0, 2)$	Yes
$F_{11} = (3, 0, 1, 1)$	$I_{11} = (4, 0, 2, 2)$	I_{11} is feasible	$X_9 = (4, 2, 3, 2, 0, 2)$	No, $X_9 > X_8$
$F_{12} = (3, 0, 2, 0)$	$I_{12} = (4, 0, 3, 0)$	Unsatisfy $W = (4, 4, 3, 2, 3, 2)$	-	-

4. Summary

Market demand may be unmet when the deterioration of generally perishable commodities is taken into account. Therefore, this paper constructs a SFDN model with deterioration consideration to deal with this situation that the commodities may be damaged during transportation. In the SFDN, each route has several available capacities with corresponding probabilities and each route has a damage rate. Based on MPs, we reference a generally SFDN evaluation algorithm to measure the SFDN reliability which represents the probability of the SFDN to successfully transport commodities from single supplier to multiple markets to meet the market demand subject to a specific level of perishability. The algorithm can be divided into two phases: the first phase is to obtain all (D, P) -MCVs and the second phase is using the RSDP approach to calculate the SFDN reliability, i.e., the probability of the union set of the (D, P) -MCVs. Through a simple distribution network, this study indicates the applicability of the algorithm.

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