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Joint Discounting and Ordering Decisions for Online Flash Sale

Danyang Shi*, Zijiao Niu

School of Business, Nanjing Normal University, Nanjing 210023, China

*dyshi0119@163.com

Abstract

Platform retailers are increasingly adopting flash sale (FS) on their online platform with the main goal of getting additional Cross-buying profit. How to set promotion price and order quantity to maximize total profit is a key challenge faced by platform retailers. Aiming at addressing this practical problem, this paper constructs a model comprising a platform retailer and many branders selling products on the online platform. Branders share a proportion of sales revenue with platform retailer. Platform retailer purchases products from one brander and will suffer losses when returning unsold FS product to the brander. The promotion price is often lower than normal selling price, which will bring platform retailer new platform users. Results show that when the Cross-buying profit is relatively high, the online platform can adopt online FS. And the higher unit Cross-buying profit is, the lower optimal promotion price and order quantity will be; When the other conditions are the same, the platform retailer should choose product with low price elasticity; In addition, the higher new user development rate and Revenue-sharing rate are, the lower promotion price and order quantity will be.

Keywords

Flash Sale; Discounting; Order Quantity; Cross-buying Profit.

1. Introduction

As customers become accustomed to online shopping, online flash sale (FS), i.e., selling limited products at a considerable discount within a specified time, has gained significant popularity in recent years. Witnessing Vente-privee's phenomenal growth since its inception, many platform retailers increasingly provide FS on their online platform, such as Alibaba, JD, Suning and so on. Through providing branders an opportunity to dispose excess inventory and increase sale volumes, platform retailers earn sales revenue shared by branders.

At present, in addition to get shared-revenue, many platform retailers start to use online FS to earn additional Cross-buying profit, which comes with no additional cost[1]. For example, Intime Department Store. Although the number of customers with Cross-buying behavior is relatively small, the total Cross-buying revenues are really considerable. In order to achieve this main objective, platform retailers purchase products from branders and improve the previous online FS from the following two aspects: First, choosing best selling commodities of famous branders as FS products instead of unsold or obsolete products. Second, setting a much shorter selling period, usually about one hour, rather than one day or more. Moreover, the quantity of these FS products is pretty small. This time-limited and availability-limited promotion method creates a feeling of scarcity, stimulating customers to visit online platforms much more frequently [2-3]. Based on this, customers are more likely to purchase other products on the online platform, which will generate Cross-buying profits to platform retailers.

A key challenge online platform retailers faced with is to set promotion price and order quantity so as to maximize total profit. On the other hand, if the promotion price is too low, the sales profit of FS product will be reduced, and thus the obtained Cross-buying profit may not be enough to make up for the reduced sales profit; If the promotion price is too high, the number of customers will reduce, and thus the Cross-buying profit will be relatively low. On the other hand, if the order quantity is too small, it may lead to customer complaints, which will have an adverse impact on the online retail platform; If the order quantity is too large, online retail platform will suffer losses when returning unsold FS products to branders. Furthermore, Cross-buying profit differs from different products, in order to maximize total profit, platform retailers will face with a challenge of choosing product.

In order to address these practical problems, we consider a setting composed of a platform retailer and many branders selling products on the online platform. Branders share a proportion of sales revenue with the platform retailer. In order to earn Cross-buying profit, the platform retailer purchases products from one brander and then sell these products on the platform by adopting online FS. When FS products aren't sold out, the platform retailer will suffer losses when returning unsold products to the brander. Under this scenario, we construct a model to study the optimal decisions of the platform retailer. Numerical analysis is provided to show the impacts of model related parameters on the platform retailer's optimal strategies and total profit.

Our contribution to the literature is twofold. First, we extend the results of previous studies by addressing joint pricing and ordering decisions of platform retailer when adopting online FS to obtain Cross-buying profit. The online FS and Cross-buying profit have been studied many times, but these two factors have never been studied simultaneously. Second, since the promotion price is often lower than normal selling price, which will attract many new platform users, therefore, we take new user development rate into consideration.

The rest of this paper is organized as follows. In Section 2, a brief literature review is presented. Section 3 introduces the model and studies the platform retailer's optimal decisions. Numerical analysis can be found in Section 4. Section 5 summaries the main conclusions. To ease exposition, we present the proofs of our findings in the Appendix.

2. Literature Review

Our study is related to three fields of previous literature: online FS, customers' Cross-buying behavior and joint pricing and ordering decisions. In this section, we will review the relevant literature briefly.

There has been growing literature on online FS. Many researchers found that firms can benefit from adopting online FS. Najjar argued that online FS can build excitement, increase the number of new users and encourage them to visit the websites more frequently and then make purchase[4]. Customers visiting FS websites often buy more than one time compared with other customers. Huang and Benyoucef pointed out that online FS can help firms to build brand loyalty, increase sales and move surplus inventory quickly[5]. Dilme and Li argued that the seller can benefit from holding online FS because online FS can create scarcity for future customers, allowing the seller earn more[6]. Optimal decisions when adopting online FS have also been studied. For instance, Ferreira et al. studied demand forecasting and price optimization of a retailer under online FS[7]. Zhang et al. constructed a model to study the brander's optimal discounted price and sale quantity when selling a new product on the online FS platform[8]. There are also some literature investigating customer behavior under online FS. Eisenbeiss et al. found that the evaluation of online FS by consumers not only depends on the discount level and the time constraint, but also relates to the purchase situation and the type of the promoted product[9]. Zhang et al. explored the impact of strategic consumers on competing retailers adopting advance selling to sell a new product and demonstrated that selling new products on FS platform can mitigate the impact of strategic customers on retailers selling new products by adopting advance selling[10]. All above extant researches investigate online FS mainly from the perspective of customers or branders using online FS platform. Our work differs from these papers in two aspects. First, we study online FS from the view of the platform retailer, who not only provides online FS but also uses online FS. Second, the main objective of the platform retailer using online FS is to get Cross-buying profits, which is common in real-world business practice but not studied before.

Customers' Cross-buying behavior has received much of attention recently. Cross-buying refers to customer's practice of buying additional products and services from the same provider in addition to the ones he or she currently has[11]. Many existing researches have shown the positive aspects of customers' Cross-buying behavior, such as increasing purchase frequency and customer profitability [12-16]. In contrast, Shah et al. argued that customers with persistent adverse behavior traits tend to engage in unprofitable Cross-buying[17]. Drivers of customers' Cross-buying behavior have also been studied, such as Verhoef et al., Verhoef and Donkers, Soureli et al. and Evanschitzky et al. [18-21]. In addition, Gao and Su, Zhang et al. and Yang and Zhang studied the omni-channel operations considering Cross-buying profits[22-24]. Our work differs from these papers in that, we investigate that in order to achieve the main objective of getting more Cross-buying profit and maximal total profit, how should the platform retailer set the discount level and order quantity.

There is an extensive literature on joint pricing and ordering decisions. These researches primarily concentrate on how to find an optimal selling price and order quantity to maximize the expected profit. Whitin and Mills firstly studied joint pricing and ordering problems in a single-period model[25-26]. Petruzzi and Dada summarized previous works and addressed this problem using an additive and multiplicative demand function, in which they substituted "stocking factor" for the ordering decision[27]. Liu and Van Ryzin, Cachon and Swinney, Liu and Shum, and Ozer and Zheng analyzed the impact of strategic consumer behavior on firm's joint pricing and rationing decisions[28-31]. Wei and Choi, Chiu and Choi, and Yan et al. studied the joint pricing and ordering strategies based on risk aversion[32-34]. Many researchers study joint pricing and ordering based on specific product, such as perishable product and fashion product[35-39]. Different from previous literature, we consider the normal selling price as an exogenous decision and study the joint discounting and ordering decisions in a single-period model for online FS.

3. Model Formulation and Solution

Consider a setting in which there is an online platform retailer (hereafter "he") and many branders. Branders selling products on the online platform pay a proportion λ of their sales revenue to the platform retailer. When determining to adopt online FS, the platform retailer firstly chooses one product of one brander (hereafter "brander M") and orders a quantity of q at the wholesale price ω. Simultaneously, the platform retailer decides promotion price p_o, the online FS selling date and selling period. Consumers can preview what is going to be promoted and detailed information on the platform before online FS selling period. As soon as the selling period arrives, consumers can purchase FS products. FS products are off the shelves when the preset time ends or all items are sold out, whichever comes first. When the selling time ends, if FS products are not sold out, the platform retailer will return these remaining FS products to the brander and receive $\delta\omega$ for each product, where δ is refund factor.

In this study, we assume that each customer can only buy one unit of FS product. The lower promotion price will not only attract a large number of existing platform users, but also attract non-platform users to register as new users. Assuming that k is the new user development rate, that is the number of new users attracted by each unit price reduction. Therefore, the demand function of FS product can be defined as

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$$\begin{aligned} d &= (a - bp_o) + k(p - p_o) + \epsilon \\ &= a + kp - (b + k)p_o + \epsilon \\ &= y(p_o) + \epsilon \end{aligned} \quad (1)$$

where a is FS product's potential market size, b is FS product's price elasticity, ϵ is a random variable with a uniform distribution defined on the range $[A, B]$, $F(\cdot)$ and $f(\cdot)$ represent the cumulative distribution function and probability distribution function of ϵ respectively. Without losing generality, we assume that $A < 0 < B$ and $E(\epsilon) = 0$. In order to ensure FS product's demand not negative, let $A > -a - kp$.

When customers browse FS product, they usually browse other products on the platform and buy them. Assuming that Cross-buying profit brought by each customer purchasing other products of brander M is θ_o (referred to unit Cross-buying profit), and Cross-buying profit that platform retailer can obtained from each customer buying other products of other branders is θ_1. Notations used in this paper are summarized in Table1.

Table 1. Notations

notation	meaning
p_o	FS product's promotion price, $0 < p_o < p$, the platform retailer's decision variable
q	FS product's order quantity, the platform retailer's decision variable
p	FS product's normal selling price
ω	FS product's wholesale price, $0 < \omega < p$
δ	refund factor
a	FS product's potential market size
b	FS product's price elasticity
k	new user development rate, $k < b$
d	FS product's demand
z	ordering factor
θ_o	Cross-buying profit brought by each customer purchasing other products of brander M
θ_1	Cross-buying profit that platform retailer can obtained from each customer purchasing other products of other branders
λ	Revenue-sharing rate, and $\lambda \in (0,1)$
π	Online platform retailer's total profit

In this paper, the operating cost of platform retailer is regarded as sunk cost, which is not considered in the model. Since the main purpose of online FS is to obtain Cross-buying profit, there is no shortage cost. On this basis, the total profit of online platform retailer when adopting online FS can be described as

$$\begin{aligned} \pi &= p_o \min\{d, q\} - \omega q + \lambda\theta_o d + \theta_1 d + \delta\omega(q - d)^+ \\ &= p_o \min\{d, q\} - \omega \min\{d, q\} + \lambda\theta_o d + \theta_1 d - (1 - \delta)\omega(q - d)^+ \\ &= (p_o - \omega)q - (p_o - \delta\omega)(q - d)^+ + (\lambda\theta_o + \theta_1)d \end{aligned} \quad (2)$$

where $(x)^+ = \max\{x, 0\}$. The first and second items in $Eq.(2)$ are the sales profit of FS product, the third and fourth items are the Cross-buying profit brought by customers when they purchase other products on the platform. The fifth item is the refund when platform retailer returns unsold product to brander M, due to $0 < \delta < 1$, platform retailer loses $(1 - \delta)\omega$ for each unsold FS product.

Following Petruzzi and Dada [27], we define an ordering factor $z = q - y(p_o)$, if $z > \epsilon$, FS products will not be sold out, which will cause losses to the platform retailer; If $z < \epsilon$, the order quantity is smaller than demand. Hence, the problem of determining the promotion price p_o and order quantity q is turned into the problem of choosing the promotion price p_o and ordering factor z. Substituting $q = z + y(p_o)$ into Eq.(2), the platform retailer's optimization problem can be written equivalently as:

$$maxE(\pi_n) = (p_o - \omega)[z + y(p_o)] - (p_o - \delta\omega)\Delta(z) + (\lambda\theta_o + \theta_1)y(p_o) \quad (3)$$

where $\Delta(z) = \int_A^z (z - x)f(x)dx$. Let the superscript $*$ denotes optimality. Proposition1 shows the optimal pricing and ordering decisions when platform retailer adopts online FS.

Proposition 1 If $\theta_l < \theta_o \leq \theta_h$, the optimal promotion price $p_o^* = \frac{I(z)}{2(b+k)} - \frac{\lambda\theta_o}{2}$, the optimal ordering factor z^*=$F^{-1}(\frac{p_o^*-\omega}{p_o^*-\delta\omega})$. According to the definition of ordering factor z, the optimal order quantity $q^* = z^* + a + kp - (b+k)p_o^*$, where $\theta_l = \frac{I(z)}{\lambda(b+k)} - \frac{2p}{\lambda}$, $\theta_h = \frac{A+a+kp}{\lambda(b+k)} - \frac{(\omega+\theta_1)}{\lambda}$, $I(z) = a + kp + z - \Delta(z) + (\omega - \theta_1)(b+k)$.

Corollary 1 In order to maximize total profit, platform retailer can adopt online FS when the unit Cross-buying profit is greater than the threshold θ_l.

Proposition1 and Corollary1 demonstrate that when the unit Cross-buying profit is relatively low, that is, $\theta_o < \theta_l$, adopting online FS may not increase platform retailer's total profit, in this case, platform retailer should not adopt FS. When the unit Cross-buying profit is greater than threshold θ_l, platform retailer should adopt online FS to attract more customers, in this case, Cross-buying profit can make up for the reduced sales profit.

Corollary 2 When platform retailer adopts online FS, for threshold θ_l:

(1) $\frac{\partial\theta_l}{\partial b} < 0$, the smaller price elasticity is, the higher threshold θ_l will be;

(2) $\frac{\partial\theta_l}{\partial k} < 0$, the smaller new user development rate is, the higher threshold θ_l will be;

(3) $\frac{\partial\theta_l}{\partial\lambda} < 0$, the higher Revenue-sharing rate is, the lower threshold θ_l will be.

Corollary 2(1) reflects that the threshold increases with the decreasing price elasticity. The intuition behind this result is that under the same discount level, the smaller price elasticity is, the less demand will decrease. For products with low price elasticity, platform retailer can adopt online FS when the Cross-buying profit is relatively high so that Cross-buying profit can make up for the reduced sales profit; Corollary 2(2) illustrates that the threshold increases with the decreasing new user development rate. This is straightforward because under the same discount level, the lower new user development rate, the less new customers will be attracted to register. Only adopting online FS when the Cross-buying profit is relatively high could increase platform retailer's total profit; An observation from Corollary 2(3) is that the threshold decreases with increasing Revenue-sharing rate. The higher Revenue-sharing rate is, the more Cross-buying profit platform retailer will obtain, even adopting online FS when Cross-buying profit is relatively low, platform retailer will be more motivated to adopt online FS.

Corollary 3 When platform retailer adopts online FS, for the optimal promotion price p_o^*:

(1) $\frac{\partial p_o^*}{\partial\theta_o} < 0$, the greater Cross-buying profit is, the lower optimal promotion price will be;

(2) $\frac{\partial p_o^*}{\partial b} < 0$, the greater FS product's price elasticity is, the lower optimal promotion price will be;

(3) $\frac{\partial p_o^*}{\partial k} < 0$, the greater new user development rate is, the lower optimal promotion price will be;

(4) $\frac{\partial p_o^*}{\partial \lambda} < 0$, the higher Revenue-sharing rate is, the lower optimal promotion price will be.

Corollary 3(1) indicates that the optimal promotion price decreases with increasing unit Cross-buying profit. This is evident that when the Cross-buying profit is relatively high, the more customers being attracted, the more Cross-buying profit platform retailer will have, therefore, platform retailer has more incentive to reduce the price to attract more customers; According to Corollary 3(2), the greater price elasticity is, the more sensitive customers will be to price promotion, setting lower promotion price will attract more customers, which will increase platform retailer's total profit; Corollary 3(3) implies that when the new user development rate is relatively high, setting lower promotion price could attract more new platform users, which will increase demand and platform retailer's total profit. Furthermore, the more new platform users being attracted, not only can improve platform retailer's total profit in the short term, but also is beneficial in the long term; Similar to Corollary 2(3), the higher Revenue-sharing rate is, the more Cross-buying profit platform retailer will get from each customer. The greater demand is, the more beneficial it is to platform retailer. Therefore, platform retailer has more incentive to set lower promotion price to attract more customers.

4. Numerical Analysis

In this section, we present numerical analysis to further illustrate the impacts of the model related parameters on the platform retailer's optimal decisions and total profit, and to derive managerial insights from analytical results. Parameters are set as follows: $a = 300$, $p = 10$, $\omega = 7$, $\delta = 0.2$, $A = -100$, $B = 100$, $\theta_1 = 4$.

4.1. Impact of the Unit Cross-buying Profit θ_o

The aim of platform retailer adopting online FS is to obtain Cross-buying profit, therefore, platform retailer's optimal pricing and ordering decisions depends on the Cross-buying profit. Assuming that price elasticity $b = 10$, new user development rate $k = 4$, Revenue-sharing rate $\lambda = 0.2$. Based on the parameters we set above, Figure 1 shows the optimal decisions and maximal total profit with respect to unit Cross-buying profit.

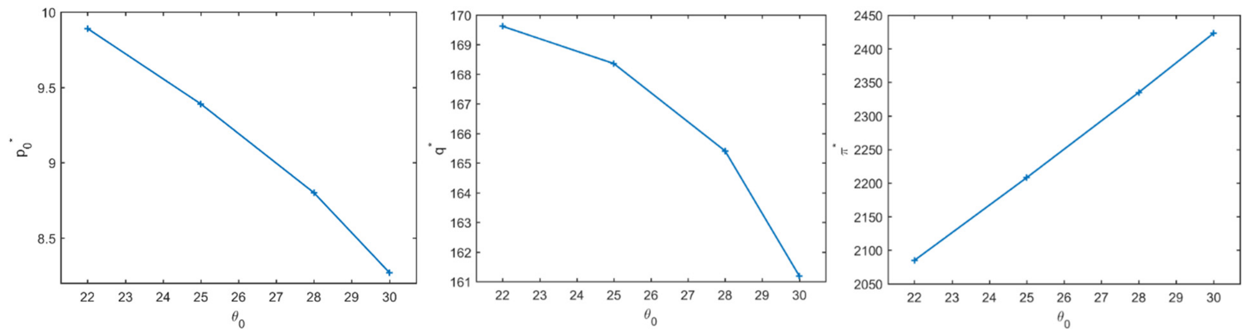

Fig 1. The effect of θ_o on the optimal decisions and total profit

Figure 1 indicates that when the unit Cross-buying profit is relatively high, that is, $\theta_o > \theta_l = 21.29$, platform retailer adopts online FS, and the optimal promotion price decreases with the increasing unit Cross-buying profit, which is consistent with Corollary 3 (1). With increasing unit Cross-buying profit, the optimal order quantity will decrease, platform retailer's total profit will increase. Therefore, unit Cross-buying profit has an influence on the optimal decisions and total profit when platform retailer adopts online FS. In order to maximize total

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profit, with the increasing unit Cross-buying profit, platform retailer should set lower promotion price and order quantity.

4.2. Impact of the Price Elasticity *b*

Since Cross-buying profit differs from different products, platform retailer has to choose the kind of FS product. Assuming that new user development rate k =4, Revenue-sharing rate λ =0.2, using b to represent the kind of online FS product and assuming b =10 and 11. Table 2 and Figure 2 show unit Cross-buying profit threshold, the optimal decisions and maximal total profit with respect to price elasticity b.

Table 2 indicates that the smaller price elasticity is, the greater unit Cross-buying profit threshold will be, which is consistent with Corollary 2 (1). Figure 2 depicts that under the same unit Cross-buying profit, the larger price elasticity is, the smaller promotion price will be, which is consistent with Corollary 3 (2). Furthermore, under the same unit Cross-buying profit, the optimal order quantity and total profit decrease with the increasing price elasticity. This is evident because with other parameters fixed, the smaller price elasticity is, the greater demand and total profit will be. Therefore, with the same condition, in order to maximize total profit, platform retailer should choose product with smaller price elasticity, set larger promotion price and order quantity.

Table 2. The effect of price elasticity b, new user development rate k and Revenue-sharing rate λ on unit Cross-buying profit threshold θ_l

b	θ_l	k	θ_l	λ	θ_l
10	21.29	3	25.61	0.15	28.38
11	14.20	4	21.29	0.2	21.29

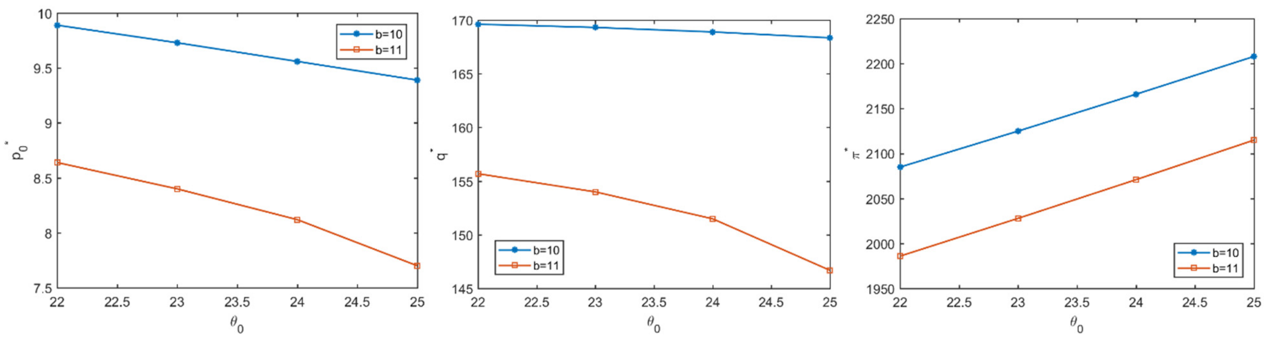

Fig 2. The effect of b on the optimal decisions and total profit

4.3. Impact of the New User Development Rate *k*

When platform retailer adopts online FS, lower promotion price could attract more new platform users, therefore, this section studies the effect of new user development rate on the unit Cross-buying profit threshold, optimal decisions and total profit. We set b = 10, λ =0.2, k =3 and 4, Table 2 and Figure 3 show unit Cross-buying profit threshold, the optimal decisions and maximal total profit with respect to new user development rate.

In Table 2, it can be found that the smaller new user development rate is, the greater unit Cross-buying profit threshold will be, which is consistent with Corollary 2(2). According to Figure 3, under the same unit Cross-buying profit, the larger new user development rate is, the lower optimal promotion price will be, which is consistent with Corollary 3(3). Furthermore, under the same unit Cross-buying profit, the optimal order quantity decreases and the total profit increases with the increasing unit Cross-buying profit. This is because that with other parameters fixed, the larger new user development rate is, the greater demand and total profit will be. Therefore, under the same condition, in order to maximize the total profit, the greater new user development rate is, platform retailer should set lower promotion price and order quantity.

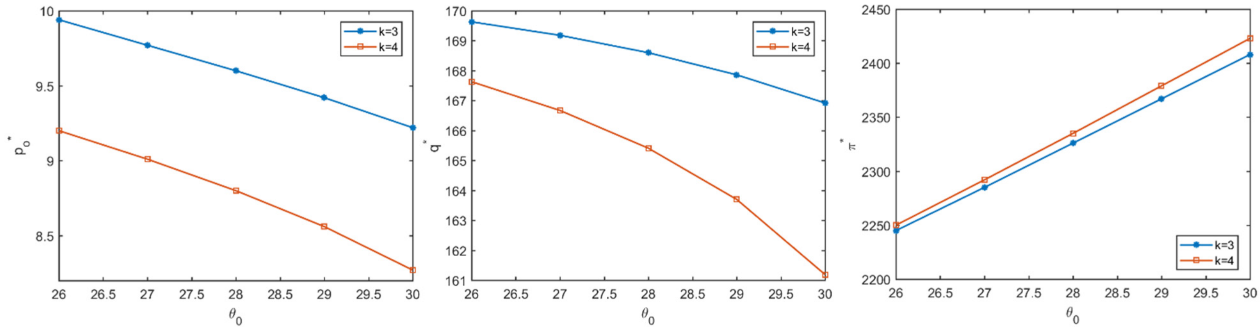

Fig 3. The effect of k on the optimal decisions and total profit

4.4. Impact of the Revenue-sharing Rate λ

Since branders share a proportion of Cross-buying profit with platform retailer, therefore, this section focuses on the effect of Revenue-sharing rate on the unit Cross-buying profit threshold, optimal decisions and total profit. Assuming that b =10, k =4, λ =0.15 and 0.2, Table 2 and Figure 4 show unit Cross-buying profit threshold, the optimal decisions and maximal total profit with respect to Revenue-sharing rate.

Table 2 shows that the greater Revenue-sharing rate is, the smaller unit Cross-buying profit threshold will be, which is consistent with Corollary 2(3). According to Figure 4, under the same Cross-buying profit, the greater Revenue-sharing rate is, the lower optimal promotion price will be, which is consistent with Corollary 3(4). Furthermore, under the same unit Cross-buying profit, the optimal order quantity decreases and the total profit increases with the increasing Revenue-sharing rate. Therefore, the greater Revenue-sharing rate is, platform retailer should set lower promotion price and order quantity.

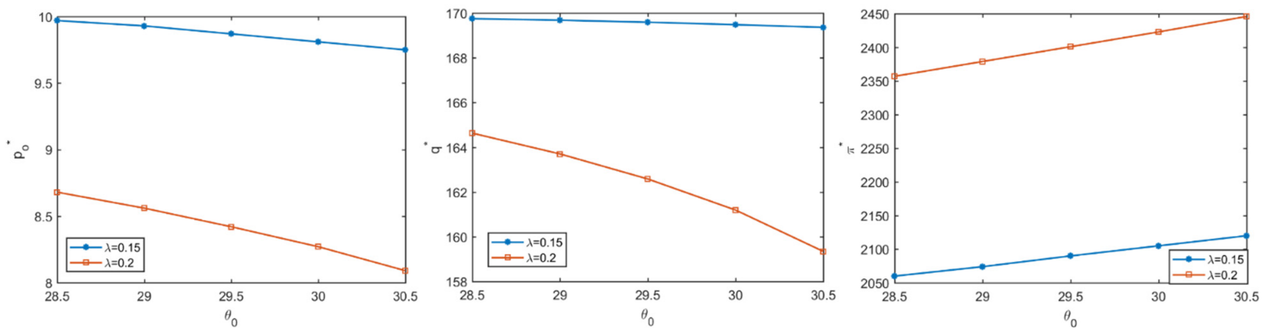

Fig 4. The effect of λ on the optimal decisions and total profit

5. Conclusion

Online FS is increasingly prevalent in recent year, especially in e-commerce industry. Inspired by many platform retailers using FS on their online platform to get Cross-buying profit, we construct a mathematical model to study how should the platform retailer set promotion price and order quantity to maximize total profit. We provide guidelines to the platform retailer on whether it should adopt online FS, which type of product it should choose, and help it make the optimal pricing and ordering decisions when adopting online FS. Our major findings in this study are summarized as follows: First, when the Cross-buying profit is relatively high, the online platform can adopt online FS. And the higher unit Cross-buying profit is, the lower optimal promotion price and order quantity will be;Second, when the other conditions are the same, the platform retailer should choose product with low price elasticity; Third, the higher new user development rate and Revenue-sharing rate are, the lower promotion price and order quantity will be.

Despite the results obtained in this paper, valuable extensions can be extended in several possible directions. For example, there are many branders selling products on platform retailer's platform and in practice, the platform retailer adopts online FS every day with different branders, therefore, future research can consider multi-branders instead of only one brander in this paper. Additionally, online FS may also have a negative impact on customer's Cross-buying behavior.

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Appendix

Proof of proposition1 For any given $z \in [A, B]$, the first order partial differential of function $\pi_n(p_o)$ is given as $\frac{\partial \pi_n(p_o)}{\partial p_o} = a + kp + z - \Delta(z) - 2(b+k)p_o + (\omega - \lambda\theta_o - \theta_1)(b+k)$, the second-order partial differential is given as $\frac{\partial^2 \pi_n(p_o)}{\partial {p_o}^2} = -2(b+k) < 0$, therefore, we prove that for any given $z \in [A, B]$, $\pi_n(p_o)$ is concave in p_o. Letting $\frac{\partial \pi_n(p_o)}{\partial p_o} = 0$, we have $p_o^* = \frac{I(z)}{2(b+k)} - \frac{\lambda\theta_o}{2}$, where $I(z) = a + kp + z - \Delta(z) + (\omega - \theta_1)(b+k)$. Since p_o satisfies $0 < p_o < p$, we have $\frac{I(z)}{\lambda(b+k)} - \frac{2p}{\lambda} < \theta_o < \frac{I(z)}{\lambda(b+k)}$.

Next, we can find the optimal ordering factor z^* when substituting $p_o^*(z)$ for p_o in Eq.(3). The first order partial differential of function $\pi_n(p_o^*(z), z)$ is given as $\frac{\partial \pi_n(p_o^*(z),z)}{\partial z} = [p_o^*(z) - \omega] - [p_o^*(z) - \delta\omega]F(z) = [p_o^*(z) - \omega][1 - F(z)] - (1-\delta)\omega F(z) = \frac{1}{2(b+k)}\{[a + kp + z - \Delta(z) - (\omega + \lambda\theta_o + \theta_1)(b+k)][1 - F(z)] - 2(1-\delta)\omega(b+k)F(z)\}$, letting $H(z) = [a + kp + z - \Delta(z) - (\omega + \lambda\theta_o + \theta_1)(b+k)][1 - F(z)] - 2(1-\delta)\omega(b+k)F(z)$, we have $\frac{\partial H(z)}{\partial z} = [1 - F(z)]^2 - [a + kp + z - \Delta(z) + (\omega - \lambda\theta_o - \theta_1)(b+k) - 2\omega(b+k)]f(z) - 2(1-\delta)\omega(b+k)f(z)$, $\frac{\partial^2 H(z)}{\partial z^2}\Big|_{\frac{\partial H(z)}{\partial z}=0} = -3[1 - F(z)]f(z) < 0$, therefore, $H(z)$ is a unimodal or monotone function. Since $H(B) = -2(1-\delta)\omega(b+k) < 0$, letting $H(A) = A + a + kp - (\omega + \lambda\theta_o + \theta_1)(b+k) \geq 0$ could guarantee the unique solution z^* of $H(z) = 0$, that is $\theta_o \leq \frac{A+a+kp}{\lambda(b+k)} - \frac{(\omega+\theta_1)}{\lambda}$, therefore, we have $z^* = F^{-1}(\frac{p_o^* - \omega}{p_o^* - \delta\omega})$.

Proof of Corollary1 According to proposition1, $\theta_l = \frac{I(z)}{\lambda(b+k)} - \frac{2p}{\lambda}$, therefore, $\frac{\partial \theta_l}{\partial b} = -\frac{I(z)}{\lambda(b+k)^2} < 0$; $\frac{\partial \theta_l}{\partial k} = -\frac{I(z) - p(b+k)}{\lambda(b+k)^2} < 0$; $\frac{\partial \theta_l}{\partial \lambda} = -\frac{I(z) - 2p(b+k)}{\lambda^2(b+k)} < 0$.

Proof of Corollary2 According to proposition1, $p_o^* = \frac{I(z)}{2(b+k)} - \frac{\lambda\theta_o}{2}$, therefore, $\frac{\partial p_o^*}{\partial \theta_o} = -\frac{\lambda}{2} < 0$; $\frac{\partial p_o^*}{\partial b} = -\frac{z + a - \Delta(z) + kp}{2(b+k)^2} < 0$; $\frac{\partial p_o^*}{\partial k} = \frac{p - (z + a - \Delta(z) + kp)}{2(b+k)^2} < 0$; $\frac{\partial p_o^*}{\partial \lambda} = -\frac{\theta_o}{2} < 0$.