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Study on the Yield of Gold and Bitcoin Portfolio Investment based on Quantitative Trading Decisions

Yajuan Wang, Pan Zhang, Liying Yao

Statistics and Applied Mathematics, Anhui University of Finance and Economics, Bengbu, 233030, China

Abstract

Aiming at the trading strategy of gold and bitcoin portfolio investment, firstly, the paper selects the price data of gold and bitcoin trading from September 13, 2016 to September 10, 2021, and constructs the asset price prediction model using grey theory to predict the future price of the two kinds of investment.Secondly,the threshold value is set to build a quantitative transaction decision model, and the nested cycle algorithm is used to obtain the optimal decision. The effect of investment strategy and the sensitivity to cost are verified. It is found that the strategy is highly sensitive to transaction cost, and transaction cost affects the transaction decision through transaction expense ratio and investment expense ratio.The prediction effect of quantitative decision model is better, which can be used for reference for other portfolio assets investment.The results show that this model can overcome the disadvantages of external factors and has high accuracy, saving time for decision-makers and greatly improving decision-making efficiency.

Keywords

Gray Prediction; Curve Fitting; Quantitative Trading; Gold; Bitcoin; Decision Strategy.

1. Introduction

Gold and Bitcoin, as two representative currencies in the real and virtual world [1], are favored by many investors for their strong preservation and appreciation capabilities[2]. In transactions in the financial market, investors tend to pursue the goal of maximizing returns, while returns and risks often coexist. Smart investors will choose to diversify risks in different fields and different products in order to achieve maximum returns[3].Depends on the traders trading strategy formulation of product market optimism, when predicting the future financial product value increase or decrease in more than a certain standard, you can buy or sell a percentage of the product, when the price range of the two will continue to hold, from this set of trading strategy determines the how much of the benefit and risk size. This paper focuses on how to predict the future price trend and make the best investment strategy based on the available data.

This kind of problem is essentially a portfolio problem. There are many previous researches on portfolio. the traditional portfolio model uses variance of expected return rate to measure risk and is based on a series of assumptions. Modern research on portfolio theory takes the lead with mean-variance model. In 1952, Markowitz published Portfolio Selection in a financial magazine, which proposed the mean-variance model for the first time and was the beginning of modern portfolio theory[4]. This paper, for the first time, studies the problem of portfolio selection from the perspective of the relationship between risk and return of assets, that is, minimization of risk under fixed return or maximization of return under certain risk [5]. In 2002, Konno et al. further discussed the optimization problem of large-scale portfolio by combining factor model and linear programming model [6]. Most of the previous studies did

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not consider the transaction cost, but all the friction in the transaction process can be reflected by the transaction cost, and its role in the price decision is the starting point and focus of the research [7]. This paper studies the design of investment strategy under the condition that the initial investment amount is determined and transaction costs are considered to optimize the daily investment portfolio[8].

2. Data Sources and Model Assumptions

The data in this paper are from question C of the 2022 American College Students Mathematical Contest in Modeling. In order to facilitate the research, the following research conditions are proposed: (1) Assume that the literature and conclusions cited in the paper are correct and reliable;(2) Assume that all cash is used up in each transaction and only one asset transaction is carried out;(3) Assume that only the data of trading days shared by bitcoin and gold are selected in data processing;(4) Assume that traders fully trust the decision model and investors make decisions only according to decision instructions[9]; (5) Assume that traders are risk-averse and sell assets when expected return rate rises to a certain extent and when expected return rate falls to a certain extent [10].

3. Predictions for Gold and Bitcoin Prices

3.1. Research Idea

First, the asset closing price sequence is established based on the closing price data, and the level ratio of the sequence is calculated.Secondly, the grey forecasting idea is used to establish the asset price forecasting model without considering the closing days[11].Finally, with five days as a cycle, the curve fitting method is used to make the comparison curve between the predicted price of gold and the real value, and the comparison curve between the predicted price and the real value of bitcoin. The curve fitting is carried out between the predicted price and the actual price to judge whether the prediction is effective.

3.2. The Research Methods :Grey Prediction

Suppose that the closing price sequence of the asset n days before is $\mathrm{x}^{(0)}=(\mathrm{x}^{(0)}(1),\mathrm{x}^{(0)}(2),\cdots,\mathrm{x}^{(0)}(n))$. Generating sequence (1-AGO):

$$\mathrm{x}^{(1)}=(\mathrm{x}^{(1)}(1),\mathrm{x}^{(1)}(2),\cdots,\mathrm{x}^{(1)}(n))=(\mathrm{x}^{(0)}(1),\mathrm{x}^{(0)}(1)+\mathrm{x}^{(0)}(2),\cdots,\mathrm{x}^{(0)}(1)+\cdots+\mathrm{x}^{(0)}(n));$$

The stage ratio of the calculated sequence is:

$$\lambda(k)=\frac{\mathrm{x}^{(0)}(k-1)}{\mathrm{x}^{(0)}(k)},\ k=2,3,\cdots,n.$$

According to the principle of grey prediction, if all the order ratios cannot all fall into the tolerable coverage $\Theta=(e^{-\frac{2}{n+1}},e^{\frac{2}{n+2}})$, appropriate constants are taken for translation transformation, and the order ratios obtained after translation transformation can all fall into the tolerable coverage [12];If all the grade ratios can fall into the tolerable coverage, the asset price prediction model GM (1,1) is established with $x^{(0)}$ as the original data[13]:

$$x^{(0)}(k)+az^{(1)}(k)=b,k=2,3,\cdots,n.$$

Where $z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1)$.

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The forecast price can be obtained from the prediction model:

$$\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}}, k=0,1,\cdots,n\text{-}1,\cdots$$

In reality, the price of asset trading market is always changing, so it is difficult to achieve accurate long-term prediction, and the predicted value is easy to deviate from the real value.In order to improve the accuracy of prediction, short-term prediction method is adopted in practical calculation[14].Taking September 11, 2016 as the first day, ignoring the days when the market was closed, and taking five consecutive days as a cycle, the asset price prediction model can predict the price of gold (or bitcoin) to the sixth day. When the trading market is closed on the sixth day and the closing price is known, the asset prices from the second day to the sixth day can be predicted to the asset prices on the seventh day, and so on. The asset prices on September 10, 2021 can be predicted by the closing price from September 5 to 9[15].

3.3. The Results of the Study

The comparison between the predicted gold or Bitcoin price and the real value is shown in Figures 1 and 2.

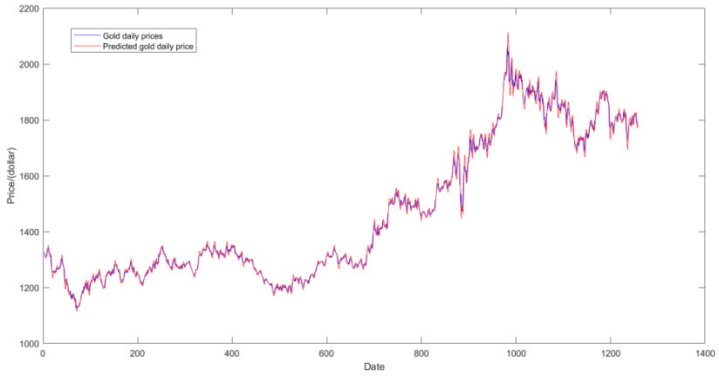

Figure 1. Predicted value and real value of gold price

As can be seen from Figure 1, gold price fluctuates up and down, showing an overall upward trend with a small range of price changes. In the five years from September 11, 2016 to September 10, 2021, the peak of gold price occurred around the 1,000th day, and the actual value and forecast value were very close, and the prediction model fitted well[16][17].

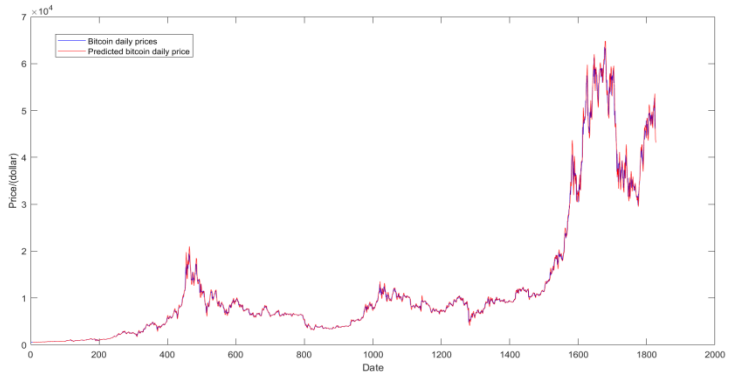

Figure 2. Predicted value and real value of Bitcoin price

As can be seen from Figure 2, although the price of Bitcoin was in a depressed state in the first four years, it suddenly increased sharply in the fifth year, reaching a peak of around $60,000,

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and then fluctuated up and down. The trajectory of the real value and predicted value of bitcoin price is very close, and the prediction model fits well[18].

4. Gold and Bitcoin Trading Strategies

4.1. Research Idea

First, the expected return rate of assets is obtained according to the predicted price, and different trading orders are obtained by comparing the predicted return rate with the asset transaction expense ratio, so as to construct a quantitative portfolio asset trading decision model, that is, when the predicted return rate is greater than the asset expense ratio, a buy order is issued.When the forecast rate of return is less than the negative asset expense ratio, issue a sell order;When the forecast yield falls between the two, issue a hold order.Secondly, the circular decision scheme of "buy - sell - buy" is adopted to judge the orders issued by the decision model.Finally, the nested loop is used to calculate the optimal trading strategy and the final asset value[19].

4.2. Research Method :Trading Strategy Model

According to the forecast price $\hat{x}^{(1)}(k+1)$, it can be known that the forecast return rate of asset $\widehat{R_{k+1}}$ on the k+1 trading day is

$$\widehat{R_{k+1}} = \frac{\hat{x}^{(1)}(k+1) - x^{(0)}(k)}{x^{(0)}(k)} * 100\% \quad (k = 0,1,\cdots,n\text{-}1,\cdots)$$

According to the formula of asset predicted return rate, the curve of predicted return rate of gold and Bitcoin over time is calculated and drawn, as shown in Figure 3.

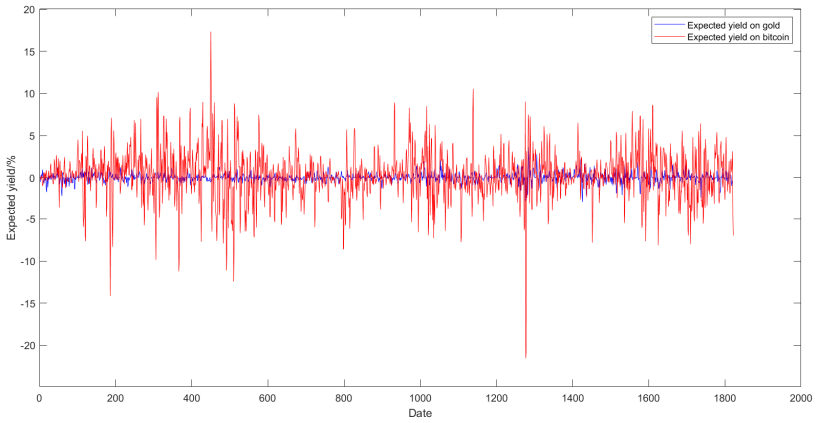

Figure 3. Expected yield of gold and bitcoin

The curve of predicted returns for gold and Bitcoin over time is shown here. In the figure, the yield curve of gold fluctuates less, and the yield curve of bitcoin fluctuates more, indicating that the risk of investing in gold is less, while the risk of investing in bitcoin is higher. According to the actual closing price data of gold and bitcoin from September 11, 2016 to September 10, 2021, although the price of gold fluctuates, it generally tends to be stable, while the price of bitcoin surges in the later period, so the predicted value is in line with the actual phenomenon[20]. In order to develop the optimal strategy, which ultimately maximizes total assets, it is necessary to minimize the number of transactions, thereby reducing commissions, and thus reducing costs[21]. In addition, thresholds need to be set to ensure low risk and to buy or sell once the yield exceeds the threshold to achieve maximum final gain[22][23].In order to make trading orders at an appropriate time, the trading decision function is constructed:

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$$I(\widehat{R_{k+1}}) = \begin{cases} 1, & \widehat{R_{k+1}} > p \\ 0, & -p \le \widehat{R_{k+1}} \le p \\ -1, & \widehat{R_{k+1}} < -p \end{cases} \quad (k = 0,1,\cdots,n\text{-}1,\cdots)$$

Where,1, 0 and -1 respectively represent buying, holding and selling, and p represents the asset transaction expense ratio. It can be known from the title that the transaction fee rates of gold and Bitcoin are 1% and 2%. In order to avoid excessive transaction frequency consuming transaction fees, the value of p can be appropriately adjusted to p gold = 1.1%, p bitcoin = 2.1%[24].

According to the asset price prediction model, if the gold yield rate of $\widehat{R_{k+1}}$ >1.1% on the k+1 trading day, the gold yield rate can be regarded as rising, and the trading system will issue an order to buy at the opening on the K +1 day. When the predicted gold yield of $\widehat{R_{k+1}}$ < −1.1%, the gold yield can be regarded as falling, and the trading system will issue an order to sell; The remaining states are judged to be held. Bitcoin's trading strategy is the same[25].

Table 1. Gold forecast price and trading decision

Trading day	Real price of gold	Gold forecast price	Forecast gold yield	orders
9/12/2016	1324.6	1324.6	/	/
9/13/2016	1323.65	1324.674	5.62E-05	0
9/14/2016	1321.75	1318.965	-0.00353914	0
.....	.....	.....	.....	.....
4/19/2017	1279.05	1294.63	0.012259676	1
4/20/2017	1282.1	1281.373	0.001816269	0
4/21/2017	1281.85	1279.549	-0.00198991	0
.....	.....	.....	.....	.....
9/7/2021	1802.15	1827.569	0.00327661	0
9/8/2021	1786	1806.718	0.002534581	0
9/9/2021	1788.25	1775.511	-0.00587311	0
9/10/2021	1794.6	1770.562	-0.00989137	0

Table 1 shows the forecast results and trading decisions of gold price from September 13, 2016 to September 10, 2021.Actually adopt "buy - sell - buy" cycle decision scheme. Suppose you spend all your cash on the same asset every time you buy it, meaning you can only hold one of cash, gold, or Bitcoin at a time. It should be noted that if you have only cash but no gold or bitcoin in hand, directly ignore the sell order -1 and wait for the buy order 1 to execute the buy decision. After buying assets, they only have one of gold or bitcoin in hand and no cash. They can only ignore the following buy order and wait for sell order -1 to sell all assets again. The cash received after each sale is w_{t+1} for the next round of trading.

$$w_{t+1} = \frac{w_t * (1-p)}{x_{buy}} * x_{sell} * (1\text{-p}), \ t = 1,\cdots,1817;$$

Which, t ranges from 1 to 1817, corresponding to September 19, 2016, to September 10, 2021, w_t represents the cash before buying the asset, x_{buy} represents the price of buying the asset, x_{sell} represents the price of selling the asset, both of which can be obtained from the forecast price. Since all cash is used to buy one asset at a time, the party with the first buy order in bitcoin and gold buys first and then buys the asset with the first buy order after selling. If the same buy

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order is placed on gold and bitcoin on the same day, an asset with a higher forecast return after fees is bought first [26].

4.3. Results Analysis

According to this method, the nested cycle is used to calculate the trading strategy and the initial investment value of $1000 on September 10, 2021 as $1650.4 [27]. The specific trading strategy results are shown in Table 2, and a more intuitive image is drawn, as shown in Figure 4.

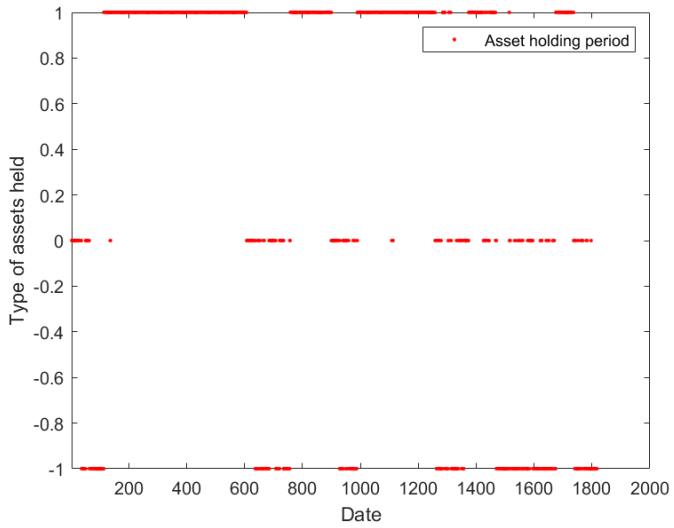

Figure 4. Asset holding period

Table 2. Trading timing table

gold		bitcoin					
buy	sell	buy	sell	buy	sell	buy	sell
113	134	36	49	1264	1270	1581	1584
137	606	63	112	1273	1274	1597	1622
758	899	637	644	1276	1277	1628	1642
990	1108	653	663	1280	1284	1644	1649
1114	1258	668	684	1293	1303	1653	1665
1284	1293	708	720	1315	1332	1671	1675
1306	1312	735	755	1336	1339	1742	1744
1375	1425	929	939	1351	1357	1746	1750
1432	1436	950	952	1471	1514	1753	1762
1446	1467	960	974	1519	1533	1770	1781
1514	1515	976	979	1536	1545	1785	1797
1675	1737	981	987	1562	1578	1799	1818

Through the construction of quantitative trading decision model, the portfolio asset investment strategy of gold and bitcoin can be calculated. This model also has important reference significance for the investment of other asset portfolios.

5. The Rationality of the Trading Strategy

5.1. Results Analysis

Two indexes, decision accuracy and backtracking accuracy, are set to represent the accuracy of the system issuing trading orders and the market sensitivity of whether the system can capture trading timing.Then, based on the obtained buy, hold and sell data, the decision accuracy and

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backtracking accuracy of gold and bitcoin are calculated respectively.Finally, according to the prediction effect of the numerical size analysis model strategy of the two accuracy indexes, high decision accuracy means that the system has a high accuracy of issuing instructions, and high backtracking accuracy means that the system can grasp the market transaction opportunities for many times.

5.2. Research Methods: Decision Accuracy and Backtracking Accuracy

In this paper, decision accuracy Pr and backtracking accuracy Re can be used to verify the effect of investment strategy quantitatively.

$$\Pr = \frac{d_{sell} + d_{buy}}{l_{sell} + l_{buy}}$$

$$\mathrm{Re} = \frac{d_{sell} + d_{buy}}{l_{up} + l_{down}}$$

Where, on the day when d_{sell} and d_{buy} trading orders are issued, The Times of gold price rising and falling according to expectations, l_{sell} and l_{buy} respectively represent The Times of sell and buy orders issued by trading system, l_{up} and respectively represent The Times of daily rise and fall of gold market exceeding the threshold.Decision accuracy Pr can be considered as the accuracy of trading orders issued by the system, while backtracking accuracy Re can be understood as the ability of the system to capture trading opportunities when they appear.

5.3. Results Analysis

Table 3. Transaction orders and actual market performance statistics of gold and Bitcoin

Statistical matrix			Predicted results			
			buy	hold	sell	total
Real results	gold	rise	8	331	6	345
		fall	4	376	6	386
		flat	0	327	0	327
		total	12	1034	12	1058
	The currency	rise	18	159	18	195
		fall	19	223	19	261
		flat	0	1	0	1
		total	37	383	37	457

The trading orders of gold and bitcoin in the five years from September 11, 2016 to September 10, 2021 and the real ups and downs of the corresponding market are calculated, as shown in Table 2.In the gold market, the trading system issued 12 buy orders in five years, corresponding to an actual close of eight days up, four days down, and no flat times. A total of 12 sell orders were issued, corresponding to an actual close of 6 days up, 6 days down, and no flat times. Therefore, the decision accuracy of the gold market can be obtained as follows:

$\Pr_{gold} = \frac{8+6}{12+12} = 0.583$,Backtracking accuracy is: $\mathrm{Re}_{bitcon} = \frac{8+6}{345+386} = 0.019$.

For the bitcoin market, 37 buy orders were issued in the five years, corresponding to an actual close of 18 days up, 19 days down and zero flat times.A total of 37 sell orders were issued, which corresponded to an actual close of 18 days up, 19 days down and zero flat times.Therefore, the decision accuracy of the gold market can be obtained as follows:

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$\Pr_{gold} = \frac{18+19}{37+37} = 0.500$,Backtracking accuracy is: $\mathrm{Re}_{bitcon} = \frac{18+19}{195+261} = 0.081$.

The decision model has high decision precision, but it can't fit the fluctuation range accurately. As a result, when the system actually issues trading orders, the accuracy is usually high; When there are trading opportunities in the market, the system will not issue trading orders when the predicted value does not reach the threshold due to the insufficient fitting of the rising and falling ranges. As a result, trading opportunities will be missed, resulting in low backtracking accuracy. Through the analysis of the prediction/backtracking accuracy of the asset return curve, we believe that the overall prediction effect of the model decision is good, and has certain investment practice value.

6. Sensitivity Analysis based on Transaction Cost

6.1. Research Idea

First, using the idea of control variables, only one parameter is changed, and the other parameters are kept equal[28]. Then, the quantitative trading decision model is used to simulate and calculate the variation of the output value of the model when the parameters change.The asset transaction expense ratio is set at a range of +0.25%, and the gold transaction expense ratio and bitcoin transaction expense ratio remain unchanged respectively. Multiple transaction expense ratios are set, and each transaction expense ratio corresponds to a final profit value, thus obtaining multiple final profit values.Secondly, according to the final return value, the curve fitting method is used to analyze the correlation between the transaction expense rate of gold and bitcoin and the final return.In addition, the correlation between gold and bitcoin transaction fee ratio and transaction strategy is analyzed, and the fitting effect is judged according to the residual graph[29].

6.2. Research Method:Simulation and Curve Fitting

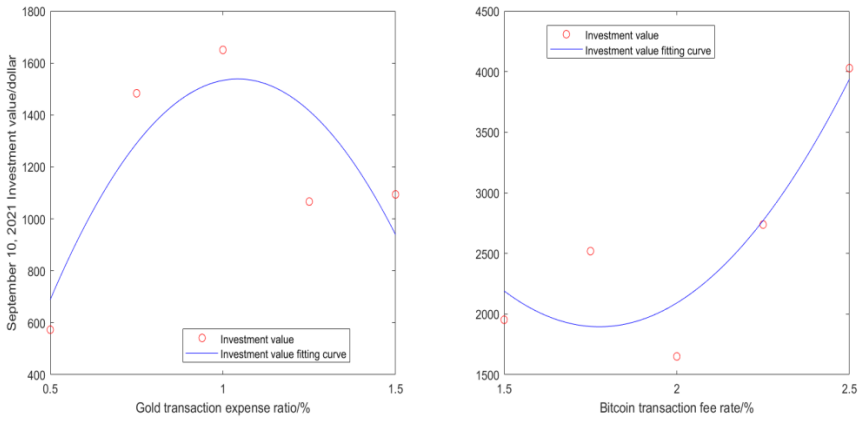

Figure 5. Correlation between gold and Bitcoin transaction expense ratio and final returns

In order to analyze the sensitivity of this strategy to transaction costs, only one parameter is changed and the other parameters remain unchanged. The quantitative transaction decision model is used to simulate and calculate the change of the model output value when the parameters change. When calculating, the asset transaction expense ratio takes a change range of +0.25%, adopts the method of control variables, keeps the size of gold transaction expense ratio and Bitcoin transaction expense ratio unchanged, and changes other parameters., for example, when the gold rate unchanged, the currency exchange rate + 0.25% change, for a number of different currency trading fee, trading in quantitative decision model, for each currency transaction fee to be able to get a decision scheme, each decision scheme can get a final solution yields numerical,Multiple final returns are calculated.Correlation analysis is made

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on the final return value and transaction expense rate of multiple Bitcoin assets, so as to obtain the influence of bitcoin transaction cost on decision making and results.Similarly, if bitcoin transaction expense ratio remains unchanged and gold transaction expense ratio changes by +0.25%, the correlation analysis result between the final return value and transaction expense ratio of multiple gold assets can be obtained, so as to analyze the influence of gold transaction cost on decision making and results[30].

For the two parameters of bitcoin transaction expense ratio and gold transaction expense ratio, if one parameter is fixed, the variable transaction expense ratio of the other asset changes with a range of 0.25%, and the regression image of asset expense ratio and final asset price under corresponding circumstances is made, as shown in Figure 5. The figure shows that the transaction expense ratio of the two assets is non-linear correlated with the final asset value. As the transaction expense ratio of gold increases, the final asset value increases gradually. When the transaction expense ratio increases to a certain value, the final asset of gold decreases. As the transaction fee rate of Bitcoin increases, the final value of bitcoin first decreases and then increases [31].

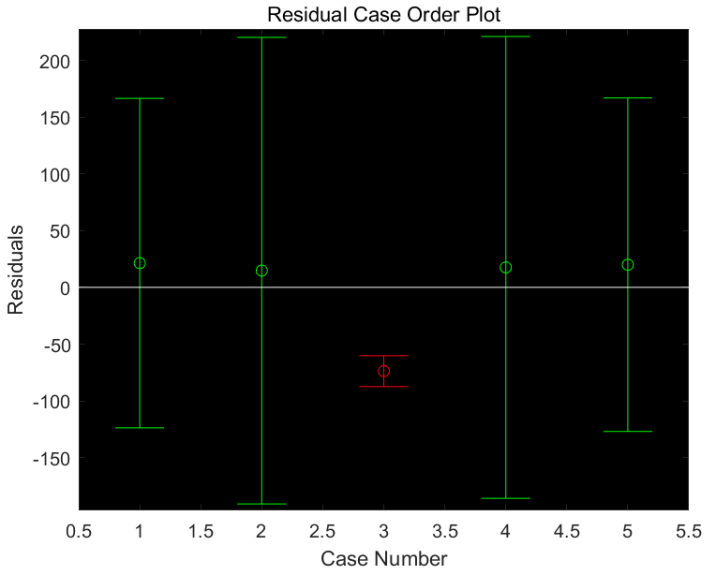

Figure 6. Trading strategy residuals

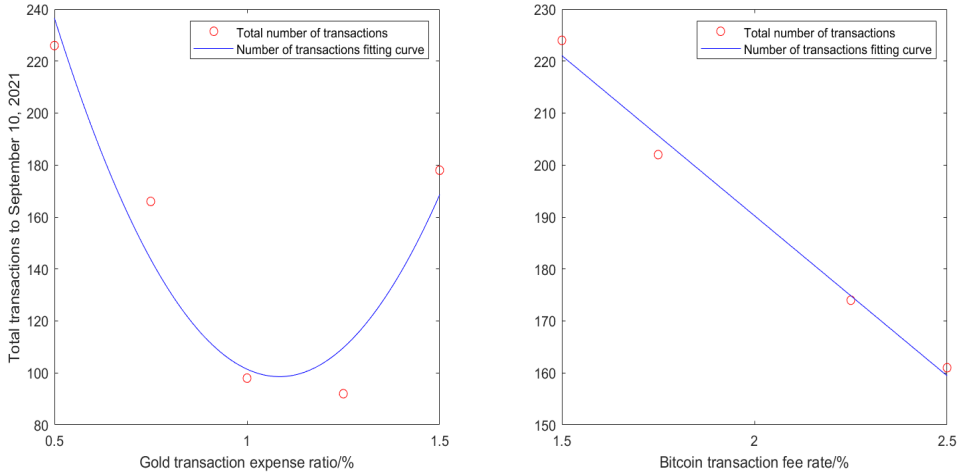

Figure 7. Correlation between gold and Bitcoin transaction fee rate and total transaction times

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When bitcoin transaction expense ratio is variable and gold transaction expense ratio is fixed, a residual chart is made for the number of transactions in the transaction strategy, as shown in Figure 6[32]. A few outliers appear in the figure, indicating that there are a few values deviating from the fitted graph line in the regression fitting. However, the range of outliers is within the acceptable range and has little influence on regression fitting.

Fixed one of the two parameters of the transaction expense ratio of gold and Bitcoin can change the transaction cost ratio of the other asset by +0.25%, and generate a regression image of the asset expense ratio and the number of transactions in the corresponding case, as shown in Figure 7.The figure shows that the transaction expense ratio of gold is non-linear correlated with the number of transactions, while the transaction expense ratio of bitcoin is negatively correlated with the number of transactions[33]. With the increase of gold transaction expense ratio, the number of transactions gradually decreases. When the transaction expense ratio increases to a certain value, the number of transactions of gold decreases[34]. The data of the quadric regression fitting model and its correlation coefficients are shown in Table 4.

Table 4. Regression model of gold and Bitcoin transaction expense ratio and final asset value and trading strategy

Relationship	Fitting model	The correlation coefficient
Gold transaction expense ratio and final asset value	y=-1592.5-2875.6x^2+6001x	0.7
Bitcoin transaction fee rate and ultimate asset value	y=14164.4+3892.1x^2-13820.4x	0.809
Gold transaction fee ratio and trading strategy	y=574+404.6x^2-877.1x	0.92
Bitcoin transaction fee rate and transaction strategy	y=313.45-61.6x	0.99

The large correlation coefficient value of the quad-fitting indicates that the fitting effect is good, and the transaction decisions and final returns of gold and bitcoin are highly sensitive to the cost, i.e., the transaction expense ratio.Transaction costs affect transaction strategies and results from the following aspects:(1) Transaction costs affect the final asset value through the respective transaction expense rates of gold and Bitcoin; (2) Transaction cost affects the total transaction strategy through investment expense ratio.Among them, the correlation coefficient between bitcoin transaction expense ratio and trading strategy is the highest, indicating that the higher the bitcoin transaction expense ratio is, the more unfavorable it is to invest.

7. Conclusion

This paper predicts the prices of gold and bitcoin based on the idea of grey prediction, and builds a quantitative trading decision model by setting thresholds. If the return rate fluctuates within the threshold range, the asset will continue to be held; if the return rate exceeds the threshold range, buy or sell orders will be issued under different circumstances.Secondly, the optimal decision and the final profit are calculated by the nested loop algorithm. Then, the effect of investment decision is verified quantitatively by calculating decision accuracy and backtracking accuracy.Finally, the sensitivity of transaction strategy to cost is obtained by simulation, curve fitting and correlation analysis.

Developing a model for reference can help investors make correct trading decisions, reduce the decision time, effectively avoid the risks caused by market uncertainty, and realize the maximization of returns under established risks or the minimization of risks under determined

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returns.The quantitative trading decision model constructed in this paper considers the influence of cost on decision-making strategy and result, overcomes the shortcoming of the previous mean-variance model that does not consider the influence of external factors such as transaction cost, has high accuracy, and reduces the decision-making time of traders, and improves the decision-making efficiency to a large extent.

As a quantitative trading decision model, the model simplifies the complex investment process into a numerical form, which is more objective and clear, and can help investors to predict the investment direction well. In addition, it can accurately predict the price changes in the future time when the market changes, so as to issue the correct trading orders, which can capture the trading opportunity to a certain extent and realize the risk minimization under the condition of stable income.

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