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# Exact Algorithm for Travel Salesman Problem with Mutiple Drones

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## Abstract

**The heuristic algorithms for travel salesman problem with mutiple drones may not prove the quality of a feasible solution, hence, an exact approach based on L-shaped decomposition is proposed. A mixed-integer linear programming model with considering limited range and capacity of drones is proposed to minimize the completion time of all vehicles. Besides, a L-shaped based decomposition with the T-reduction to strengthen both optimality and feasible cuts is designed. Finally, the feasibility and effectiveness of the proposed approach is verified through the standard data set used in the literature. The results show that the L-shaped based decomposition method outperforms the Gurobi in both solution quality and computational time. Furthermore, the acceleration technique can improve the convergence greatly.**

## Keywords

**Truck-Drone Team Logistics; Mixed-Integer Linear Programming Model; Exact Approach.**

## 1. Introduction

Drone delivery has attractive advantages in urban logistics distribution such as low cost, high speed, low pollution, and flight without ground traffic restrictions. Therefore, more and more modern logistics enterprises like SF Express, JINGdong, Cainiao, Meituan, Amazon, Google, and DHL have begun to explore and experiment a drone delivery for the last mile delivery service. However, a drone delivery also suffers two major limitations such as limited flight duration and loading capacity[1].

Many studies focused on truck-drone team logistics. It can overcome the shortcomings of drones by replenishing the cargo and energy for drones through trucks. The truck-drone team logistics can service customers either by vehicles or drones. Murray and Chu studied the truck-drone team logistics problem with a truck and a drone by extending the classic travel salesman problem (TSP)[1]. Agatz et al extended the study of Murray and Chu and proposed a traveling salesman problem with a drone (TSP-D) by allowing the truck to send and receive a drone at the same location[2]. The TSP-D is NP-hard because the TSP is NP-hard. Therefore, the TSP-D is difficult to solve. Many studies proposed a variety of heuristic algorithms or artificial intelligence algorithms to solve this problem including a genetic algorithm, an ant colony algorithm, and a simulated annealing algorithm[1-2].

Although heuristic algorithms or artificial intelligence algorithms can get a feasible solution in a relatively short time, they cannot provide a lower bound to evaluate the quality of the feasible solution. Hence, a handful of studies proposed exact algorithms for the TSP-D and its variants. For instance, Poikonen et al designed a branch and price algorithm for the TSP-D[3]. Similarly, Sebastián proposed the L-shaped decomposition algorithm to solve the TSP-D, which is an extension of the branch and cut algorithm[4].

Although TSP-D can effectively reduce logistics costs and improve distribution efficiency compared with a single truck delivery [5], the TSP-D only considers the collaborative delivery

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between a truck and a drone, which is difficult to meet the actual demand. Several studies considered the truck-drone team logistics between a truck and multiple drones (TSP-MD) and can reduce more costs than the TSP-D. However, the TSP-MD is more complex than the TSP-D due to the collaboration of multiple drones. The accurate algorithms for TSP-MD can only solve the instance with no more than 10 customers, so it is quite challenging to design an accurate algorithm for the TSP-MD.

Therefore, this study proposed an exact algorithm for the TSP-MD based on the L-shaped decomposition algorithm.

## 2. Problem Statement and Mathematical Model

Let $G=(V,A)$ be a complete graph with nodes $V=\{1\}\cup N$ and arcs $A=\{(i,j):i,j\in V\}$. Node 1 denotes the depot and $N=\{2,\ldots,n\}$ is the set of customers. A single truck is equipped with a homogeneous fleet of drones to serve all customers. Let $D$ denote the set of drones. The travel times of an arc $(i,j)\in A$ (including the service time at node $i$) for the truck and drones are $t_{ij}^{T}$ and $t_{ij}^{d}, d\in D$ respectively. In general, we assume $t_{ij}^{T}\le t_{ij}^{d}, d\in D$ according to most related literature. A drone can be carried by the truck or fly alone. The maximum flying duration of a drone is $L$. A drone can serve at most one customer alone before return to the truck because of the limited capacity of drones. Let $O\subseteq V\times N\times V$ denote the set of operations which can be performed by drones. An operation $e=(i,k,j)\in O$ shows that a drone take off from the truck at node $i\in V$, servicing node $k\in N$ and return to the truck at node $j\in V$ while the truck executes a route from node $i$ to node $j$ or waits at node $i$ (i.e., $i=j$). Multiple launch and retrieval operations are allowed to happen concurrently at a node, which requires that the truck leave a retrieval node after all drones have been retrieved at this node because of the synchronization of a truck and drones. The goal of the TSP-MD is to find a route with minimum duration including the travel and waiting time of the truck to serve all customers either by the truck or the drones while considering the synchronization between the truck and drones.

The model of the TSP-MD can be formulated as following:

$$\min \sum_{(i,j)\in A} t_{i,j}^{T} x_{i,j}^{T} + \sum_{d\in D}\sum_{(i,j,k)\in O} w_{ikj}^{d} \tag{1}$$

$$\sum_{(i,j)\in A} x_{ij}^{T} = \sum_{(j,i)\in A} x_{ji}^{T} = \gamma_i \quad \forall i\in N \tag{2}$$

$$\sum_{(i,j)\in A} x_{1j}^{T} = \sum_{(i,n+1)\in A} x_{in+1}^{T} = 1 \tag{3}$$

$$y_{ij} + y_{ji} \le \gamma_i \quad \forall i,j\in N \tag{4}$$

$$y_{ij} + y_{ji} \le \gamma_j \quad \forall i,j\in N \tag{5}$$

$$\gamma_i + \gamma_j - 1 \le y_{ij} + y_{ji} \quad \forall i,j\in N \tag{6}$$

$$f_{ij}^{d} \le x_{ij}^{T} \quad \forall i,j\in N \tag{7}$$

$$\sum_{k\in N:(i,k,j)\in O, d\in D} o_{ikj}^{d} \le y_{ij} \quad \forall i,j\in N \tag{8}$$

$$\sum_{(i,k,j)\in O, d\in D} o_{ikj}^{d} = 1-\gamma_k \quad \forall k\in N \tag{9}$$

$$\sum_{e\in\delta^+(i)} o_e^d + \sum_{j\in N} f_{ij}^d = \sum_{e\in\delta^-(i)} o_e^d + \sum_{j\in N} f_{ij}^d \quad \forall i\in N, d\in D \tag{10}$$

$$w_{ikj}^\lambda \ge \left(t_{ik}^\lambda + t_{kj}^\lambda\right)\times o_{ikj}^\lambda - t_{ij}^T \times y_{ij} \quad \forall (i,k,j)\in O, \lambda\in\Theta \tag{11}$$

$$x_{ij}^T, \gamma_j, y_{ij}, f_{ij}^\lambda, o_e^\lambda \in \{0,1\} \quad \forall i,j\in N, e\in O, \lambda\in\Theta \tag{12}$$

$$w_{ikj}^\lambda \ge 0 \quad \forall i\in N,\ \ (i,k,j)\in O, \lambda\in\Theta \tag{13}$$

The objective function minimizes the completion time including the travel and waiting time of the truck. Constraints (2) are flow conservation flow of the truck, which indicate that the truck arrives and leaves from a visited node exactly once. Constraints (3) ensure the truck leaves and returns to the depot exactly once. Constraints (4)～(6) ensure that the truck has to visit node i or j if these two nodes are in the route of the truck. Constraints (7) denote that the truck has to travel arc $(i,j)$ if drone $d\in D$ travels arc $(i,j)$ on board the truck. Constraints (8) impose that the truck has to visited node i before node j if an operation $(i,k,j)$ is executed by a drone. Constraints (9) ensure that each customer $k\in N$ is serviced exactly once either by the truck or drones. . Constraints (10) are flow conservation flow of drone $d\in D$, which indicate that drone $d$ arrives and leaves from a visited node either on board the truck or flying alone. Constraints (17) require that the truck leave a retrieval node after all drones have been retrieved at this node, where the waiting time of the truck is equal to the difference between completion time for an operation $(i,k,j)$ by drones under travel time uncertainty and the truck's travel time from node *i* to node *j*. Constraints (12)～(13) impose the domains of all variables.

## 3. L-shaped Decomposition

### 3.1. Master Problem

The L-shaped master problem (LMP) is used to determine the route of the truck and can be stated as follows:

$$\min \sum_{(i,j)\in A} t_{i,j}^T x_{i,j}^T + \theta \tag{14}$$

$$\text{Constraints (2)-(6)} \tag{15}$$

$$x_{ij}^T, \gamma_j, y_{ij} \in \{0,1\} \quad \forall i,j\in N, e\in O \tag{16}$$

### 3.2. Subproblem Problem

The goal of the L-shaped subproblem problem (LSP) is to optimality and feasibility cuts with a given solution $(x^*, \gamma^*, y^*, u^*, \theta^*)$ of the LMP. The LSP can be represented as following:

$$\min \sum_{d\in D} \sum_{(i,k,j)\in O} w_{ikj}^d \tag{17}$$

$$\text{Constraints (7)–(13)} \tag{18}$$

There are two cases when solving the LSP with a given solution to the LMP[6].

(1) The LSP is infeasible. In this case, a feasible cut is added to the LMP to eliminate the current solution.

$$\sum_{(i,j)\in A:x_{ij}^{\mathrm{T}*}=1}\left(1-x_{ij}^{T}\right)+\sum_{(i,j)\in A:x_{ij}^{\mathrm{T}*}=1}x_{ij}^{\mathrm{T}}\geq 1 \tag{19}$$

(2) The LSP has an optimal solution with the optimal objective value $W^*$. The optimality solution of original problem is obtained if $\theta^*=W^*$. While An optimality cut is added to the LMP to lift the auxiliary variable $\theta$ if $\theta^*<W^*$.

$$\theta\geq -W^*[\sum_{(i,j)\in A:x_{ij}^{T*}=1}\left(1-x_{ij}^{T}\right)+\sum_{(i,j)\in A:x_{ij}^{T*}=0}x_{ij}^{T}]+W^* \tag{20}$$

## 4. Computational Experiments

### 4.1. Test Instances and Parameters Setting

We generate a set of test instances with customer sizes for $n\in\{5,..,16\}$.The locations of nodes including depots and customers are randomly selected according to a uniform distribution over $100\times 100$ square following the same methodology as literature . The travel speeds of the truck and drones are set to 1 and 2 respectively. The drone flight duration is set to 1800 according to . The number of drones are set to $L=\infty$. All experiments are performed on a Inter I5 11300H 3.1GHz PC with 8GB RAM. The maximum CPU running time is set to be 1200 seconds.

### 4.2. Results Analysis

In order to test the computational efficiency of the proposed method, Gurobi, L-shaped and L-shaped & acceleration algorithms are respectively applied. As shown in Table 1, when the case size is larger than 12, Gurobi cannot obtain the optimal solution within 1200s (the average error is 6.5%), while both L-shaped algorithm and L & A algorithm can obtain the optimal solution. Meanwhile, the computation time of L-shaped algorithm is reduced by 89.1% compared with Gurobi, while the computation time of L & A is reduced by 94.3% compared with Gurobi. Therefore, the L-shaped algorithm and L & A are obviously superior to Gurobi, and the acceleration technology also significantly improves the convergence speed of the algorithm.

**Table 1.** Performance of different algorithms

| Algorithms | Case size | Time(s) | Gap(%) |
|---|---|---|---|
| Gurobi | 12 | 472.3 | 6.5 |
| L | 16 | 51.7 | 0.0 |
| L&A | 16 | 27.1 | 0.0 |

## 5. Conclusion

In this study, we proposed a mixed integer linear programming model for the TSP-MD and designed an accurate algorithm based on L-shaped decomposition to solve this problem. The experimental results show that TSP-MD saves the total delivery time more than TSP-D. Besides, the L-shaped algorithm is superior to Gurobi in both solving time and quality.

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