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Portfolio Returns based on Markowitz Efficient Boundary Model

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Abstract

Maximizing returns is always people's investment goal. Gold and Bitcoin have some hedging abil- ities, and their prices fluctuate greatly, making them popular for investors .We use the risk analysis of the Sharp ratio to compare with the portfolio-managed. We use Markowitz effective boundary model to select a portfolio with medium risk and high yield. then we used the time series to calculate the change difference between the final result and the standard value after the transaction cost change. A system of equations based on a number of yield values can be established to meet the optimization requirements, introduce computational historical data, and replace the relevant transaction costs with the above model to clarify the change curve of the final return results. The investment return model established in this paper can provide some reference for the relevant practitioners and has certain application significance.

Keywords

Investment Income; Markowitz' Efficient Frontier; Sensitivity Analysis.

1. Introduction

Market traders often buy and sell volatile assets with the goal of maximising their total returns. There is usually a commission on every sale. Two of those assets are gold and bitcoin. There is volatile property in the market, and its value constantly fluctuates over time [1].Traders can maximize total returns by constantly buying and selling volatile property to continuously increase the total value of the holding property. But different investment projects require a different commission for each sale. Investing is not blind; the key issue of trading decision is to execute the right decision at the right time [2]. When holding a variety of property such as gold, Bitcoin needs more strategy. Investors decide whether to buy the property or sell or continue to hold the property in the portfolio [3]. We can provide us with the current transaction optimal strategy in the daily price changes of investment property known to date, finally achieving the purpose of maximizing the corresponding returns.

To explore the best investment trading strategy, our team has studied it with gold and Bitcoin.Bitcoin can be traded every day, gold can only be traded on trading days, and they have different trading commissions [4]. Our team studied investment trading strategies by performing different portfolios for gold and bitcoin during the five-year trading period from November 9,2016 to October 9,2021 [5].

2. The Model of Markowitz' Efficient Frontier

2.1. Risk Analysis by Sharpe Ratio

The sharpe ratio is calculated as:

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$$\text{sharp ratio} = \frac{r_p - r_f}{\sigma_p} \tag{1}$$

where $r_p$is the expected portfolio, $r_f$is the risk-free rate, andσ_p is the portfolio standard deviation.

2.2. Comparison with Markowitz' Efficient Frontier

2.2.1. Markowitz's Efficient Frontier

Markowitz's Efficient Frontier which is used to manage portfolio can be expressed as the followingmathematical model:

$$\min \sigma_p^2 = X^{'} \Omega X = \sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} X_i X_j \sigma_{ij} \tag{2}$$

$$\text{s.t } r_p = \sum x_i r_i$$

$$\sum x_i = 1$$

Where $r_p$is the expected yield rate of return on portfolio investment, σ_p^2 is the variance of the yield.

This approach proposes a framework to evaluate the risk and returns of a portfolio. The line of efficient frontier shows the portfolios with the highest returns for a given risk profile. For our evaluation, we designed an agent to pick a moderate risk high reward portfolio from an efficient frontier graph calculated at every time step based on the previous 30-time steps' performance. Average returns: -1%

2.2.2. Comparison

Below is a side-by-side comparison of the 2 policies in the same environment: RL grows portfolio to 31%, Markowitz' shrinks to 96% Our model has several advantages over Markowitz:

We can see that the efficient boundary is not effective for highly volatile assets such as Bitcoin. However, during times of increased volatility or when all assets are going down, the RL decides to hedge the losses by selling the assets and increasing the cash in hand — very smart strategy when we haven't enabled a short-sell option. Markowitz' Efficient Frontier is shown as figure1.

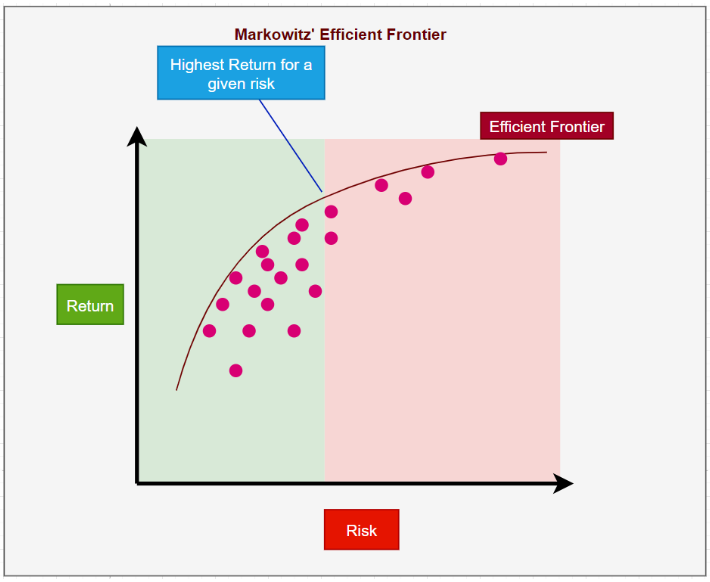

Figure 1. Markowitz' Efficient Frontier

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The traditional Markowitz model uses the average return and risk of assets in a period of time to analyze the risk and return, but the actual situation is that both investment return and investmentrisk change in a period of time

The theory assumes a single-term investment, in which capital is allocated once at the beginning of the period and nothing is changed thereafter. This is not an actual investment behavior, nor does it apply to multi-stage problems. But the RL strategy seems to be to identify bursts of smallsurges in price and capitalize on that immediately.

The classical mean-variance model is often threatened by real data. In the real financial market, the distribution of a series of returns often deviates from the normal distribution, presenting kurtosisand skewness, which makes the variance of returns unsuitable for risk measurement.

Therefore, although Markowitz's portfolio theory has laid a foundation for the development of modernportfolio research, it still has some limitations when facing practical problems. We see that **RL consistently outperforms Markowitz' approach in our experiments.**

3. Transaction Cost Time Series Model

3.1. Problem Solution

To effectively calculate the sensitivity of the transaction strategy to the transaction cost, we used the time series to calculate the change difference between the final result and the standard value after thetransaction cost change.

We aimed to find transaction costs with higher yields, with the change difference measured as:

$$\begin{cases} \sqrt{(x_{o1}-x_1)^2+(y_{o1}-y_1)^2}=r_1 \\ \sqrt{(x_{o1}-x_3)^2+(y_{o1}-y_3)^2}=r_1 \\ (x_1-x_3)^2+(y_1-y_3)^2=2r_1^2-2r_1^2cosa \end{cases} \tag{3}$$

$$\begin{cases} (z-z_1)^2+(y-y_1)^2=r_1^2 \\ (z-z_2)^2+(y-y_2)^2=r_2^2 \\ (z-x_n)^2+(z-y_n)^2=r_n^2 \end{cases} \tag{4}$$

We build a system of equations based on multiple yield values, so that they can meet the optimization requirements.

$$\begin{matrix} 2z(z_n-z_1)+2y(y_n-y_1)=z_n^2-z_1^1+y_n^2+r_1^2-r_n^2 \\ 2z(z_n-z_2)+2y(y_n-y_2)=z_n^2-z_2^2+y_n^2+r_2^2-r_n^2 \\ \cdots\cdots \\ 2z(z_n-z_{n-1})+2y(y_n-y_{n-1})=z_n^2-z_{n-1}^2+y_n^2+r_{n-1}^2-r_n^2 \end{matrix} \tag{5}$$

The derivation of the simplification can be obtained:

$$\mathrm{M}=\begin{bmatrix} 2(z_n-z_1) & 2(y_n-y_1) \\ 2(z_n-z_1) & 2(y_n-y_2) \\ M & M \\ 2(z_n-z_{n-1}) & 2(y_n-y_{n-1}) \end{bmatrix} \tag{6}$$

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$$N=\begin{bmatrix} z_n^2 - z_1^2 + y_n^2 - y_1^2 + r_1^2 - r_n^2 \\ z_n^2 - z_2^2 + y_n^2 - y_2^2 + r_2^2 - r_n^2 \\ \cdots \\ z_n^2 - z_{n-1}^2 + y_n^2 - y_{n-1}^2 + r_{n-1}^2 - r_n^2 \end{bmatrix} \tag{7}$$

$$Z = \begin{bmatrix} x \\ y \end{bmatrix} \tag{8}$$

Get its optimized location:

$$Z_{est} = (M^T M)^{-1} M^T N \tag{9}$$

If the time series$\{Y_t\}$satistied:

$$Y_t = \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \varepsilon_t$$

The partial autocorrelation function of the sample is:

$$\varphi_{kk} = \begin{cases} \rho_1 \\ \frac{\rho_k - \sum_{j=1}^{k=1} \varphi_{k-1,j} \cdot \rho_{k-j}}{1 - \sum_{j=1}^{k=1} \varphi_{k-1,j} \cdot \rho_j} \end{cases} \tag{10}$$

3.2. Comprehensive Analysis

We conducted a comprehensive analysis to obtain the optimization mode under different circumstances. We calculated the maximum return value when the transaction cost is the gold transaction cost and thebitcoin transaction cost combination as 0.005,0.01,0.02,0.02,0.04,0.05 and 0.1, respectively. Gold transaction cost 0.005, Bitcoin transaction cost 0.01is shown as figure2.

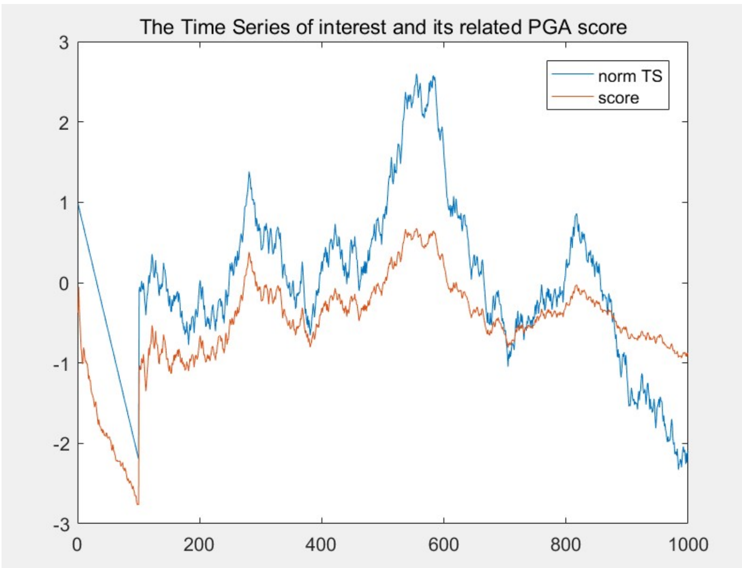

Figure 2. Gold transaction cost 0.005, Bitcoin transaction cost 0.01

In fact, in order to maximize the investment utility, we need to calculate the historical data and substitute the relevant transaction costs with the model described above, so as to clarify the change curve of the final benefit results. Gold transaction cost 0.02, Bitcoin transaction cost 0.04 is shown as figure3. Gold transaction cost 0.05, Bitcoin transaction cost 0.01 is shown as figure4.

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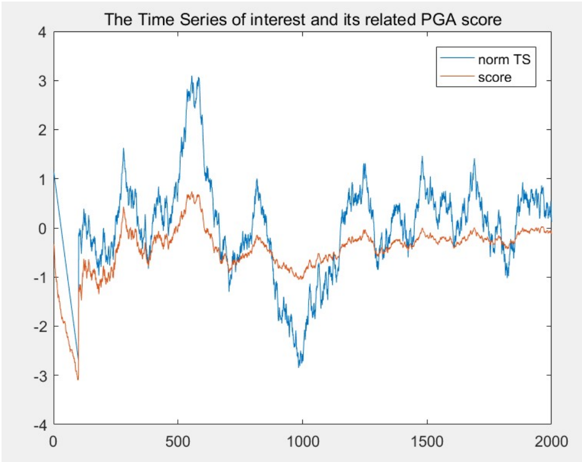

Figure 3. Gold transaction cost 0.02, Bitcoin transaction cost 0.04

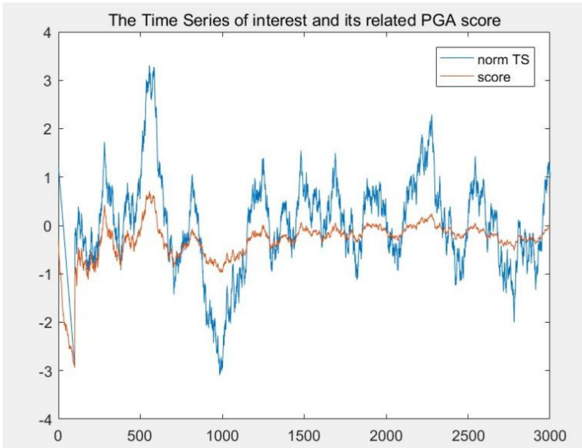

Figure 4. Gold transaction cost 0.05, Bitcoin transaction cost 0.01

Using such a model, we will get the maximum investment utility and calculate the change situationand the return maximum. As can be seen from the figure, the different transaction costs will change thelocation of their maximum value, based on the fact that the different transaction costs can be changed in the time series by affecting the investment expectations. Different transaction costs do not have much impact on the maximum, presumably because the maximum has been greatly affected by the daily pricein the past, and long transactions have stabilized it.

4. Sensitivity Analysis

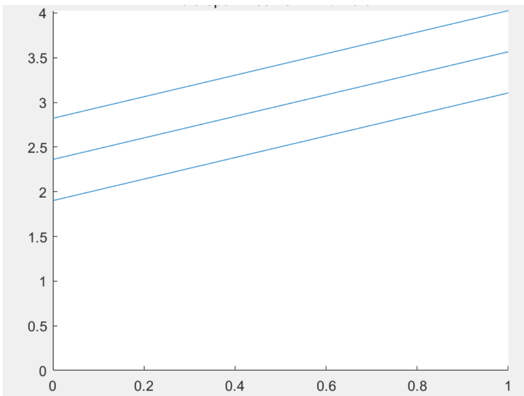

Figure 5. The relationship between gold transaction cost and total assets when keeping the bitcoin transaction cost at 0.01

In fact, we can complete the analysis of the sensitivity of the transaction strategy to the transaction costaccording to its requirements, and we determine its sensitivity analysis mode.

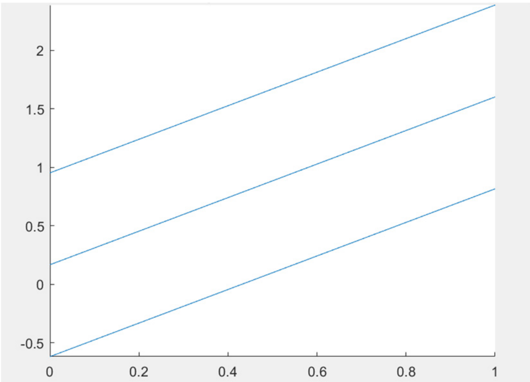

Figure 6. The relationship between bitcoin transaction cost and total assets when keeping the gold transaction cost at 0.01

Figure 5 shows the relationship between gold transaction cost and total assets when keeping the bitcoin transaction cost at 0.01; Figure 6 shows the relationship between bitcoin transaction cost and total assets when keeping the gold transaction cost at 0.01. The generation of results is moresensitive to changes in gold transaction costs. As can be seen in figures a and b, as the transaction costincreases, the total assets ultimately obtained also decline significantly. The point itself that increasingcosts make profits decrease is obvious. At the same time, our model avoids unnecessary costly wasteand chooses to remain in a wait-and-see mode, so the cost also influences the trading strategy to someextent, making the model degenerate. The rate of decline of total assets in both charts is fast and slow,but the effect shown in the graphs shows that the degree of decline of the curve is still in a relatively stable state, indicating the health of our model.

5. Conclusion

Maximizing returns is always people's investment goal. Gold and Bitcoin have some hedging abil-ities, and their prices fluctuate greatly, making them popular for investors. We use the risk analysis of the Sharp ratio to compare with the portfolio-managed. We use Markowitz effective boundary model to select a portfolio with medium risk and high yield. then we used the time series to calculate the change difference between the final result and the standard value after the transaction cost change. A system of equations based on a number of yield values can be established to meet the optimization requirements, introduce computational historical data, and replacethe relevant transaction costs with the above model to clarify the change curve of the final return results. Later, according to the needs of the trading strategy, complete the sensitivity analysis of the trans-action strategy on the trading cost (calculate the impact of variable factors on profit), determine the sensitivity analysis model.

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